Wealth Dynamics in Communities

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Abstract

This paper explores how favor exchange in communities influences investment and wealth dynamics. Our main result identifies a key obstacle to wealth accumulation: wealth crowds out favor exchange. Thus, low-wealth households forego profitable investments, since growing their wealth would entail losing access to the support of the community. The result is that some communities are "left behind," with persistently lower wealth than the rest of the economy. Using numerical simulations, we show that "place-based" policies encourage wealth accumulation and so particularly benefit left-behind communities. Kinship, friendship, or religious ties among community members can similarly spur investment and wealth accumulation.

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1 Introduction

The United States and many other countries have grappled with persistent and prevalent wealth inequality: poor communities persist even in otherwise-rich economies. Members of these "left-behind" communities suffer from low wealth and limited economic prospects (Ganong and Shoag 2017, Austin et al. 2018). Yet, despite those problems, left-behind communities are not dying out; instead, many households express a strong preference for staying in them (Bartik 2020). One reason that households choose to stay is that their neighbors are an important source of "practical and social support" (Economist 2020). In the United States, for example, left-behind communities offer "a crucial source of childcare" and other assistance (Economist 2017).

The fact that communities provide this kind of support makes it all the more puzzling that they don't catch up with wealthier areas. Community support has the potential to help households make wealth-creating investments. In practice, however, support does not translate to growing wealth; instead, households tend to merely "get by" (Warren et al. 2001), even when they have opportunities to grow their wealth by paying down high-interest debts or making other high-return investments (Ananth et al. 2007, Stegman 2007, Mel et al. 2008). For all of its benefits, why doesn't the support of the community translate to growing wealth?

This paper studies how communities shape wealth accumulation and welfare. To do so, we develop a model that combines favor-exchange in communities with investment dynamics. Our main result uncovers a key obstacle to wealth accumulation in the community: wealth crowds out favor exchange. Poor households therefore face a stark choice between growing their wealth and accessing support from their communities. The result is a persistent wealth gap between left-behind communities and the rest of the economy.

The reason for this result is what we call the "too big for their boots" effect: wealth undermines trust among community members. In particular, neighbors are willing to support a household only if they trust it to reciprocate in the future. Rather than reciprocate, households can instead leave the community; in our model, leaving consists of moving to an anonymous market that we call the "city." Wealth makes the city more attractive, so wealthy households are more tempted to leave the community rather than repay neighbors for past support. Thus, as households get wealthier, they lose the ability to access significant community support. To avoid this "too big for their boots" effect, households forego profitable investments and keep their wealth low. Thus, while community support encourages consumption and improves welfare, it also depresses investment, deepens inequality, and perpetuates low wealth.

In her seminal study of an impoverished United States community, the anthropologist Carol Stack vividly illustrates how wealth can undermine trust (Stack [1975]). Favor exchange is ubiquitous among the households that she studies, with neighbors trading childcare, clothing, transportation, and food with one another. These relationships are typically dynamic—neighbors repay one another in kind and over time—and are backed by the threat of social sanctions. Strikingly, Stack [1975] suggests that it is the wealthiest (though still poor) members of the community who are most at risk of being excluded from favor exchange:

> "Members of second-generation welfare families have calculated the risk of giving. As people say, 'The poorer you are, the more likely you are to pay back.' This criterion often determines which kin and friends are actively recruited into exchange networks." - *p. 43*

This rule of thumb might seem strange, since wealthier households presumably have more resources to trade. However, Stack [1975] points out that wealthier households are also less reliant on the community, since they can leave and meet their needs in a nearby city, Chicago.

To explore the implications of the "too big for their boots" effect, we develop a dynamic model of a household that resides in a tight-knit community and trades consumption goods with its neighbors. To compensate those neighbors, the household can make an up-front monetary payment, and it can also promise—but not commit to—an ex post, in-kind reward. The household invests its remaining wealth in a positive-return investment. In any period, the household can choose to leave this community for the city, which is more productive than the community but also anonymous.

In the city, anonymity implies that favor exchange is impossible. Thus, once a household moves to the city, it uses wealth alone to purchase goods and services. It optimally does so according to a standard consumption-investment problem, where wealth and consumption grow over time. In contrast, a household in the community can augment consumption by engaging in favor exchange with its neighbors. The household follows through on such exchanges only if it is otherwise punished. Since a wealthy household can better escape these punishments by moving to the city, it is less trusted to repay favors. This gives rise to the "too big for their boots" effect: wealthier households are less able to exchange favors with their neighbors.

Our main result builds on this logic to identify two reasons why left-behind communities remain left behind. First, wealthy households leave for the city, so that only poorer households remain in the community. This initial selection effect is then compounded by a treatment effect: households in the community under-invest and experience sharply limited long-term wealth. Indeed, for some households, under-investment is so severe that wealth actually decreases over time. Thus, while households in the city take full advantage of investment opportunities and get richer, households in the community stay poor and may even get poorer.

This result sheds light on what policies might help left-behind communities. Researchers have proposed two approaches to spurring growth in these communities: "place-based" policies provide benefits for households *so long as they remain in the community*, while "mobility-based" policies encourage households to *leave and access greater economic opportunities elsewhere* (Bartik 2020). Using numerical simulations, we show that place-based policies mitigate the "too big for their boots" effect and encourage wealth accumulation. Mobility-based policies have the opposite effect: they encourage some households to leave, but they also discourage wealth accumulation and decrease welfare for those who remain. Thus, our model provides a rationale for place-based policies. Family, religious, and social ties can similarly bind households to their communities and thereby encourage wealth accumulation.

The contribution of this paper is to explore how communities influence wealth dynamics. Since those dynamics occur in the shadow of an anonymous market, the city, we are related to papers on the interaction between formal and informal markets (Kranton 1996, Banerjee and Newman 1998, Gagnon and Goyal 2017, Banerjee et al. 2018, Jackson and Xing 2019). Unlike that literature, we focus on how this interaction shapes wealth dynamics. In so doing, we explore an intertemporal spillover from future wealth to current cooperation. This focus also separates us from other papers that study how communities distort decision-making (Austen-Smith and Fryer 2005, Hoff and Sen 2006).

Our model of exchange draws on the relational contracting literature (Macaulay 1963, Bull 1987, Levin 2003, Malcomson 2013), particularly those papers that consider the role of outside options (Baker et al. 1994, Kovrijnykh 2013). We contribute to this literature by introducing a new state variable, wealth, and by modeling anonymous exchange as an alternative to long-term relationships in the community. We are therefore related to papers that study cooperation in communities (Wolitzky 2013, Ali and Miller 2016, 2018, Miller and Tan 2018); however, we abstract from questions of network structure to instead focus on wealth dynamics.

Our main result is that favor exchange discourages investments and perpetuates inequality. Unlike papers that study under-investment due to lumpy costs (e.g., Nelson 1956, Advani 2019), we assume that investment entails no fixed costs. This type of under-investment has been documented in, e.g., Karlan et al. [2019] and Balboni et al. [2020]; the theory literature has proposed two possible explanations for it. First, individuals might be subject to time-inconsistent preferences (Bernheim et al. 2015) or temptations (Banerjee and Mullainathan 2010). Second, capital markets might be imperfect; for instance, monopolistic lenders might expropriate the returns on investment (Mookherjee and Ray 2002) or impose restrictive covenants that limit long-term investment (Liu and Roth 2019). Our model offers a different explanation, which is that households voluntarily limit investment to maintain access to community support. Thus, even though our model assumes neither behavioral preferences nor capital-market imperfections, poor households do not take full advantage of investment opportunities and have persistently low wealth.

The idea that money eases commitment problems dates to Jevons [1875]'s argument that money solves the "double coincidence of wants." Prendergast and Stole [1999, 2000] build on this idea to compare market and barter economies. We build on a related idea to explore wealth dynamics.

2 Model

A long-lived **household** ("it") has initial wealth $w_0 \geqslant 0$ and discount factor $\delta \in (0,1)$. The household initially lives in the community. At the start of each period $t \in \{0,1,...\}$, it can choose to stay in the community or irreversibly move to a city.

If the household is still in the community in period t, then it plays the following **community game** with a short-lived **neighbor** t ("she"), who represents another member of the community:

1. The household requests a consumption level $c_t \geq 0$ and offers a payment $p_t \in [0, w_t]$ in exchange. Note that p_t cannot exceed the household's wealth, w_t.

2. Neighbor t accepts or rejects this exchange, $d_t \in \{0,1\}$. If she accepts ($d_t = 1$), then she receives p_t and incurs the cost of providing c_t. If she rejects ($d_t = 0$), then no trade occurs.

3. The household decides how much of a favor to perform for neighbor t, $f_t \geqslant 0$.

4. The household invests its remaining wealth, $w_t - p_t d_t$, to generate w_{t+1}. Let $R(\cdot)$ give the return on investment, so that

$$w_{t+1} = R(w_t - p_t d_t).$$

The household's period-t payoff is $\pi_t = U(c_t d_t) - f_t$. Neighbor t's payoff is $(p_t - c_t)\, d_t + f_t$. Interactions are observed by all neighbors, reflecting the idea that the community is tight-knit.

We assume that consumption utility $U(\cdot)$ and investment returns $R(\cdot)$ are strictly increasing and strictly concave, with $U''(\cdot)$ and $R'(\cdot)$ continuous and $U(0) = 0$, $\lim_{c \downarrow 0} U'(c) = \infty$, and $\lim_{c \to \infty} U'(c) = 0$. Investment generates positive returns, and strictly so below a threshold $\bar{w} > 0$: $R(0) = 0$, $R'(w) > \frac{1}{\delta}$ for $w < \bar{w}$, and $R'(w) = \frac{1}{\delta}$ for $w \geqslant \bar{w}$.

If the household has moved to the city by period t, then it plays the **city game** with a short-lived **vendor** t ("she"). The city game is identical to the community game in all but two ways. First, each vendor observes only her own interaction with the household, so that transactions are anonymous in the city. Second, consumption has higher marginal utility, reflecting that the city is more productive at providing goods and services. Letting $\hat{U}(\cdot)$ be the household's consumption utility in the city, so that $\pi_t = \hat{U}(c_t d_t) - f_t$, we assume that $\hat{U}'(c) > U'(c)$ for all $c > 0$. Otherwise, $\hat{U}(\cdot)$ satisfies the same conditions as $U(\cdot)$. A household in the city remains there forever.[1]

The household's continuation payoff in period t is

$$\Pi_t \equiv (1 - \delta) \sum_{s=t}^{\infty} \delta^{s-t} \pi_s.$$

We characterize household-optimal equilibria, which are the Perfect Bayesian Equilibria that maximize the household's *ex ante* expected payoff.

We maintain the following assumption, which ensures that households in the community have access to strictly positive-return investments.

Assumption 1 *Define $\bar{c} > 0$ as the solution to $U'(\bar{c}) = 1$. Then, $R(\bar{w} - \bar{c}) > \bar{w}$.*

In the context of Stack [1975], the household and neighbors are members of a predominantly low-income community called "the Flats." Members of the Flats exchange a wide

[1] In Online Appendix B, we show that the "too big for their boots" effect arises even if households in the city can return to the community.

variety of goods and services (c_t), including food, clothing, childcare, and transportation. While households can pay one another with money (p_t), compensation also occurs via favor exchange. For example, the recipient of childcare ($c_t > 0$) might agree to reciprocate with future childcare ($f_t > 0$). Households accumulate wealth ($R(\cdot)$) by repaying high-interest debt and making other high-return investments. As Stack [1975] emphasizes, households in the Flats can move to a nearby city, Chicago, which harbors better opportunities ($\hat{U}' > U'$) but is far enough away that a household must leave the Flats to access them.

3 Life in the City

We first characterize wealth dynamics in the city. A household in the city faces a standard consumption-investment problem, takes full advantage of investment opportunities, and accumulates wealth.

Transactions are anonymous in the city, so $f_t = 0$ in equilibrium. Vendor t is therefore willing to accept any offer that covers his costs, $p_t \geq c_t$; consequently, every equilibrium entails $p_t = c_t$ in every $t \geqslant 0$, so that $w_{t+1} = R(w_t - c_t)$. For a household with wealth w, the resulting optimal consumption and payoff are given by:

$$\hat{C}(w) \in \arg\max_{c\in[0,w]} \Big((1-\delta)\hat{U}(c) + \delta\hat{\Pi}\left(R\left(w-c\right)\right)\Big) \text{ and}$$
$$\hat{\Pi}(w) = \max_{c\in[0,w]} \Big((1-\delta)\hat{U}(c) + \delta\hat{\Pi}\left(R\left(w-c\right)\right)\Big).$$

Our first result shows that $\hat{\Pi}(w)$ and $\hat{C}(w)$ are the unique equilibrium outcome in the city.

Proposition 1 *Both $\hat{\Pi}(\cdot)$ and $\hat{C}(\cdot)$ are strictly increasing, with $\hat{\Pi}(\cdot)$ continuous. In any equilibrium, $\Pi_t = \hat{\Pi}(w_t)$ and $c_t = \hat{C}(w_t)$ in any $t \geq 0$, with $(w_t)_{t=0}^{\infty}$ increasing and*

$$\lim_{t\to\infty} w_t \geq \bar{w}$$

on the equilibrium path.

The proof of Proposition 1 is routine and relegated to Online Appendix A. Since $R'(w) > \frac{1}{\delta}$ for $w < \bar{w}$, the standard Euler equation,

$$\hat{U}'(\hat{C}(w_t)) = \delta R'(w_t - \hat{C}(w_t))\hat{U}'(\hat{C}(w_{t+1})),\ \forall\, t, \tag{1}$$

implies that wealth increases until at least $\bar{w}$ in the city. Figure 1 simulates equilibrium consumption as a function of w and the resulting consumption and wealth dynamics.

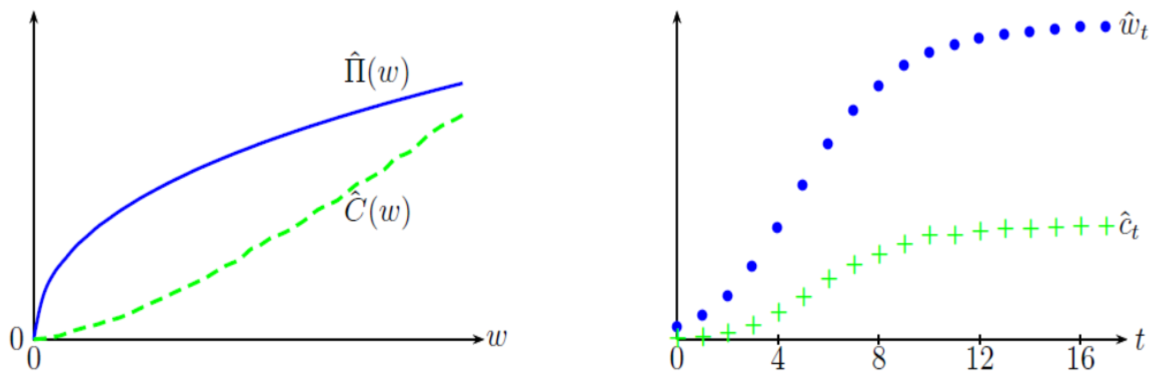

Figure 1: Left panel: the household's equilibrium payoff and consumption as a function of w; Right panel: consumption and wealth over time, starting at $w_0 = 0.108$.

4 Household-Optimal Wealth Dynamics

We now turn to our main result, which characterizes household-optimal equilibria in the community. Section 4.1 states this result and discusses its intuition. Section 4.2 gives the proof.

4.1 Life in the Community

Our main result identifies two reasons why wealth in the community remains substantially below wealth in the city. First, there is a *selection margin:* the wealthiest members of the community leave, so that only the less-wealthy households remain. Second, there is a *treatment effect:* those households that remain do not take full advantage of investment opportunities, so their long-run wealth is sharply limited.

To build intuition for this result, note that the sole advantage of the community is that neighbors can observe and punish a household who reneges on $f_t > 0$. Therefore, the household can credibly promise $f_t > 0$ to compensate neighbor t for providing consumption $c_t > p_t$. Thus, while the city has higher productivity, favor exchange is possible only in the community.

The opportunity to engage in favor exchange is most attractive to low-wealth households, which would have minimal consumption in the city. Conversely, wealthy households can consume a lot in the city and so would derive little benefit from using favor exchange to further augment consumption. This argument implies that the richest households leave the community and the poorest households stay, giving us our selection margin.

To understand wealth dynamics within the community, consider a household with wealth w_t that stays. Suppose that this household consumes c_t and invests $I_t \equiv w_t - p_t$. As in the standard Euler equation, (1), investment allows for higher future consumption, which has marginal benefit $U'(c_{t+1})R'(I_t)$. Investment also leads to lower p_t and hence lower c_t, which has marginal cost $U'(c_t)$.

In the community, this familiar cost and benefit are joined by a second, indirect cost of investment. Since the household can always renege on f_t, leave, and earn $\hat{\Pi}(R(I_t))$, the *maximum* favor that can be sustained in equilibrium depends on I_t. Denoting the household's on-path continuation payoff by $\Pi^*(R(I_t))$, f_t must satisfy the following *dynamic enforcement constraint:*

$$f_t \leq \bar{f}(I_t) \equiv \frac{\delta}{1-\delta}(\Pi^*(R(I_t)) - \hat{\Pi}(R(I_t))). \tag{2}$$

Assuming that $\bar{f}(\cdot)$ is differentiable and (2) binds, the marginal effect of investment on the equilibrium favor in period t is $\bar{f}'(I_t)$. Consumption, c_t, optimally changes one-for-one with f_t, so a change in f_t leads to a corresponding change in the household's payoff of $U'(c_t) - 1$.

In a household-optimal equilibrium, the two marginal costs of investment must equal its marginal benefit, leading to the following *modified Euler equation:*

$$U'(c_t) = \delta R'(I_t)U'(c_{t+1}) + \bar{f}'(I_t)(U'(c_t) - 1). \tag{3}$$

The "too big for their boots" effect holds whenever $\bar{f}'(I_t) < 0$, so that investment crowds out favor exchange. We will show that $U'(c_t) - 1 > 0$ for any household in the community. Consequently, whenever the "too big for their boots" effect holds, the investment that satisfies (3) is strictly below the investment that satisfies the standard Euler equation. This is the sense in which the household "over"-consumes and "under"-invests.

This intuition identifies the mechanism that leads to under-investment, but it elides a key complication: the maximum credible favor, $\bar{f}(\cdot)$, depends on both $\Pi^*(\cdot)$ and $\hat{\Pi}(\cdot)$, which in turn depend on the household's future consumption and investment decisions. Wealth is a persistent state variable that affects all of these decisions, rendering a full characterization of household-optimal equilibria intractable.

Our main result, Proposition 2, instead characterizes the selection and treatment effects. *Selection* is summarized by a wealth level, $w^{se} < \bar{w}$, such that the household leaves whenever $w_t \geq w^{se}$, and a set $\mathcal{W} \subseteq [0, w^{se}]$ such that the household stays forever whenever $w_t \in \mathcal{W}$. *Treatment* is summarized by a wealth level, $w^{tr} < w^{se}$, such that long-term wealth of a household in the community is no more than w^{tr}.

Proposition 2 *Impose Assumption 1. There exist wealth levels* $w^{tr} < w^{se} \in (0, \bar{w})$ *and a positive-measure set* $\mathcal{W} \subseteq [0, w^{se}]$ *such that in any household-optimal equilibrium:*

1. ***Selection.*** *The household stays in the community forever if* $w_0 \in \mathcal{W}$*, and otherwise leaves in* $t = 0$*.*

2. ***Treatment.*** *If the household stays in the community, then* $(w_t)_{t=0}^{\infty}$ *is monotone, with*

$$\lim_{t \to \infty} w_t \leqslant w^{tr}.$$

Moreover, $\mathcal{W} \cap [w^{tr}, w^{se}]$ has positive measure.

Section 4.2 gives the proof of this result. To see the intuition, note that we have already argued that wealthy households leave the community while some poorer households stay. Any household that stays, stays forever, since otherwise favor exchange would unravel from their last interaction within the community. This gives us a wealth level above which households leave, w^{se}, and a set $\mathcal{W}$ of poorer households that stay forever.

Now, consider a household that stays with initial wealth *just below* w^{se}. Such a household is close to indifferent between leaving and staying. Thus, if this household's *future* wealth remains near w^{se}, then $f_t \approx 0$. Since staying is optimal only if $f_t \gg 0$ in some t, the household must under-invest so severely that its wealth decreases. The proof of Proposition 2 strengthens this result by showing that $(w_t)_{t=0}^{\infty}$ is monotone and that a positive measure of households, $\mathcal{W} \cap [w^{tr}, w^{se}]$, stay in the community despite having initial wealth near w^{se}. These households experience declining wealth.

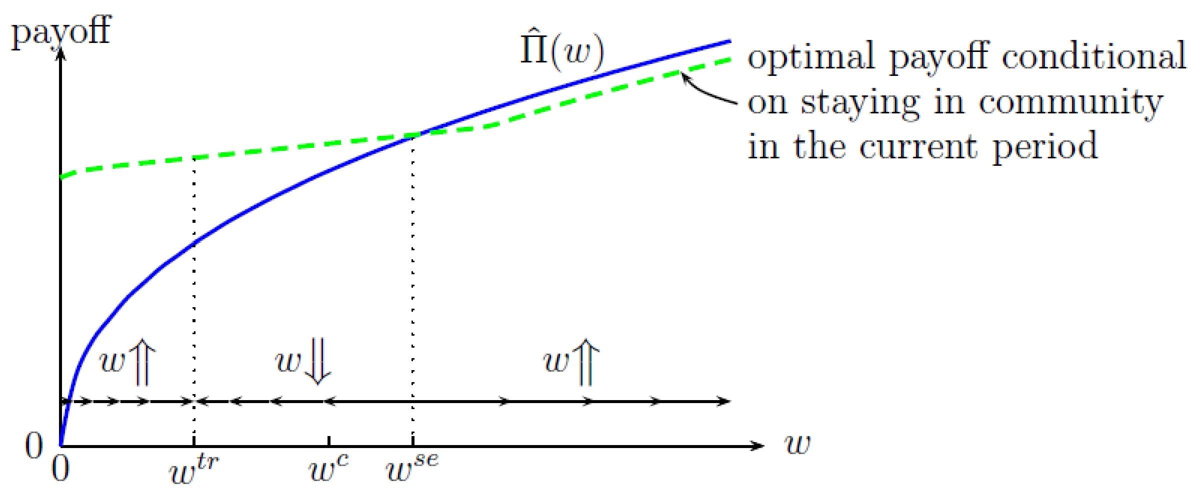

Figure 2: Simulated household-optimal equilibrium payoffs and wealth dynamics

Figure 2 summarizes Proposition 2. In this simulation, the household moves to the city if $w_0 \geqslant w^{se}$ and otherwise stays. Among those that stay, households with wealth below w^{tr} grow their wealth, but only to w^{tr}. Those with $w_0 \in (w^{tr}, w^{se})$ have declining wealth over time.

An implication of this result is that one-time transfers do not necessarily spur further investment. In Figure 2, consider a one-time transfer that increases the household's wealth. If its resulting wealth satisfies $w_t < w^{se}$, then long-term wealth remains w^{tr} and so is completely unaffected by this transfer. This result resonates with Karlan et al. [2019], which finds that low-wealth entrepreneurs receiving debt relief tend to quickly fall back into debt. In contrast, a transfer that is large enough to lead to $w_t \geq w^{se}$ *does* induce further investment, but only by spurring the household to leave the community.

4.2 The Proof of Proposition 2

Let $\Pi^*(w)$ be the maximum equilibrium payoff of a household with wealth w. Define

$$\begin{aligned}
\Pi_c(w) \equiv \quad & \max_{c \geq 0, f \geq 0} \quad \{(1-\delta)(U(c)-f) + \delta \Pi^*(R(w+f-c))\} \\
& \text{s.t.} \qquad 0 \leq c - f \leq w \qquad (4) \\
& \qquad\qquad f \leq \frac{\delta}{1-\delta}\left(\Pi^*(R(w+f-c)) - \hat{\Pi}(R(w+f-c))\right). \qquad (5)
\end{aligned}$$

We show that $\Pi_c(w)$ is the household's maximum payoff conditional on staying in the community in the current period. The household's maximum equilibrium payoff, $\Pi^*(w)$, is the maximum of $\hat{\Pi}(w)$ and $\Pi_c(w)$.

Lemma 1 *The household's maximum equilibrium payoff is* $\Pi^*(w_0) = \max\left\{\hat{\Pi}(w_0), \Pi_c(w_0)\right\}$, *where* $\Pi_c(\cdot)$ *and* $\Pi^*(\cdot)$ *are strictly increasing.*

Proof of Lemma 1: We show that $\Pi_c(\cdot)$ is the household's maximum equilibrium payoff conditional on staying in the community in the current period. In any equilibrium, neighbor 0 accepts only if $c_0 \leq p_0 + f_0$. The household's continuation payoff is at most $\Pi^*(R(w_0 - p_0))$ and at least $\hat{\Pi}(R(w_0 - p_0))$. Hence, it is willing to do favor f_0 only if

$$f_0 \leq \frac{\delta}{1-\delta}\left(\Pi^*(R(w_0-p_0)) - \hat{\Pi}(R(w_0-p_0))\right).$$

Setting $c_0 = p_0 + f_0$ yields $\Pi_c(w_0)$ as an upper bound on the household's payoff from staying.

This bound is tight. For any (c_0, f_0) that satisfies (4) and (5), it is an equilibrium to set $p_0 = c_0 - f_0 \geq 0$, play a household-optimal continuation equilibrium on-path, and respond to any deviation with the household leaving and $f_t = 0$ in all future periods.[2] Thus, $\Pi_c(\cdot)$ is the household's maximum equilibrium payoff conditional on staying. It follows that $\Pi^*(w) = \max\{\hat{\Pi}(w), \Pi_c(w)\}$. Since $\Pi_c(\cdot)$ is strictly increasing by inspection and $\hat{\Pi}(\cdot)$ is strictly increasing by Proposition 1, $\Pi^*(\cdot)$ is strictly increasing. $\square$

The next four lemmas characterize household-optimal equilibria in the community. First, we show that households that stay in the community, stay forever.

Lemma 2 *If $w_0 \geq 0$ is such that $\Pi^*(w_0) > \hat{\Pi}(w_0)$, then in any $t \geq 0$ of any household-optimal equilibrium, $\Pi^*(w_t) > \hat{\Pi}(w_t)$ on the equilibrium path.*

Proof of Lemma 2: Suppose $t > 0$ is the first period in which $\Pi^*(w_t) = \hat{\Pi}(w_t)$. Let $\{c_{t-1}, f_{t-1}\}$ achieve $\Pi_c(w_{t-1})$. Since $\Pi^*(w_t) = \hat{\Pi}(w_t)$, (5) implies $f_{t-1} = 0$. The household therefore earns a higher payoff by exiting in $t-1$ and choosing the same c_{t-1}. This contradicts $\Pi^*(w_{t-1}) > \hat{\Pi}(w_{t-1})$. $\square$

Second, we bound consumption from above in the community.

Lemma 3 *Fix $w_0 \in \{w : \Pi^*(w) > \hat{\Pi}(w)\}$, and let $\{c_t\}_{t=0}^{\infty}$ be consumption in a household-optimal equilibrium. Then $U'(c_t) \geq 1$ in all $t \geq 0$.*

Proof of Lemma 3: Consider a household in the community with wealth w_t, and suppose $U'(c_t) < 1$ in period t. If $f_t > 0$, consider decreasing f_t and c_t by $\epsilon > 0$. Doing so is feasible and increases the household's payoff at rate $1 - U'(c_t) > 0$ as $\epsilon \to 0$.

Suppose $f_t = 0$, and let $\tau > t$ be the first period after t such that $f_\tau > 0$. Consider decreasing p_t and c_t by $\epsilon > 0$, increasing p_τ by $\chi(\epsilon)$, and decreasing f_τ by $\chi(\epsilon)$, where $\chi(\epsilon)$ is chosen so that $w_{\tau+1}$ remains constant. Then, $\chi(\epsilon) \geq \frac{\epsilon}{\delta^{\tau-t}}$ because $R'(\cdot) \geq \frac{1}{\delta}$. As $\epsilon \to 0$,

[2] If $f_t = 0$ in all $t \geq 0$, then the household is willing to leave because $U \leq \hat{U}$.

this perturbation increases the household's payoff by at least $\delta^{\tau-t}\frac{1}{\delta^{\tau-t}} - U'(c_t) > 0$. It is an equilibrium because $f_s = 0$ for all $s \in [t, \tau-1]$, so (5) still holds in these periods.

We conclude that if $U'(c_t) < 1$, then $f_s = 0$ in all $s \geq t$. But then $\Pi_c(w_t) < \hat{\Pi}(w_t)$, so $\Pi_c(w_0) \leq \hat{\Pi}(w_0)$ by Lemma 2. Contradiction of $w_0 \in \{w : \Pi^*(w) > \hat{\Pi}(w)\}$. $\square$

Third, we show that wealthy households leave the community, while poorer households stay.

Lemma 4 *The set* $\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ *has positive measure. Moreover,* $w^{se} \equiv \sup\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ *satisfies* $0 < w^{se} < \infty$.

Proof of Lemma 4: First, we show that $\Pi^*(0) > \hat{\Pi}(0) = 0$. Because $\lim_{c \downarrow 0} U'(c) = \infty$, there exists a $c > 0$ such that $c \leq \delta U(c)$. Suppose that in all $t \geq 0$, $f_t = c_t = c$ and $p_t = 0$ on the equilibrium path. Any deviation is punished by $f_t = 0$ in all future periods and the household immediately exiting. This strategy delivers a strictly positive payoff. It is an equilibrium because $c \leq \delta U(c)$ implies (5). Thus, $\Pi^*(0) > 0$. Since $\Pi^*(w)$ is increasing and $\hat{\Pi}(w)$ is continuous, there exists an open interval around 0 such that $\Pi^*(w) > \hat{\Pi}(w)$. So $\left\{w : \Pi^*(w) > \hat{\Pi}(w)\right\}$ has positive measure.

Next, we show that $w^{se} < \infty$. Let $\bar{c}$ satisfy $U'(\bar{c}) = 1$. From Lemma 3, we know that $\Pi^*(w) \leq U(\bar{c})$ for every $w \in \{w : \Pi^*(w) > \hat{\Pi}(w)\}$. For any w_0 such that $R(w_0 - \bar{c}) \geq w_0$, the household can set $p_t = c_t = \bar{c}$ in every $t \geq 0$ in the city. For such w_0, $\hat{\Pi}(w_0) > \hat{U}(c) > U(c)$, so w^{se} satisfies $R(w^{se} - \bar{c}) < w_0$ and hence $w^{se} < \infty$. $\square$

Finally, household-optimal equilibria exhibit monotone wealth dynamics.

Lemma 5 *In any household-optimal equilibrium,* $(w_t)_{t=0}^{\infty}$ *is monotone.*

The (tedious) proof of Lemma 5 is relegated to Appendix A. The key step of this proof shows that household-optimal investment, $w_t - p_t$, increases in w_t. Wealth dynamics are therefore monotone: if $w_1 \geq w_0$, then $w_2 = R(w_1 - p_1) \geq R(w_0 - p_0) = w_1$ and so on, and similarly if $w_1 \leq w_0$.

We can now prove Proposition 2.

Selection: Follows immediately from Lemma 2. The set $\mathcal{W}$ has positive measure by Lemma 4. By the proof of Lemma 4, $R(w^{se} - \bar{c}) < \bar{c}$, so Assumption 1 implies that $w^{se} < \bar{w}$. $\square$

Treatment: We argue that there exists $w^c < w^{se}$ such that for any $w_t \in (w^c, w^{se})$, if the household stays in the community, then $w_{t+1} \leq w^c$. Consider $w_t < w^{se}$, and suppose $w_{t+1} > w^c$. Since $\Pi^*(\cdot)$ is increasing, (5) holds only if

$$f_t \leq \Delta(w_t) \equiv \frac{\delta}{1-\delta}(\Pi^*(w^{se}) - \hat{\Pi}(w_t)).$$

Proposition 1 implies that $\Delta(w_t)$ is strictly decreasing and continuous.

Continuity implies that $\lim_{w \uparrow w^{se}} \hat{\Pi}(w) = \hat{\Pi}(w^{se})$. If $\Pi^*(w^{se}) > \hat{\Pi}(w^{se})$, then $\Pi^*(\cdot)$ increasing and $\hat{\Pi}(\cdot)$ continuous imply $\Pi^*(w) > \hat{\Pi}(w)$ just above w^{se}, contradicting the definition of w^{se}. Therefore, $\Pi^*(w^{se}) = \hat{\Pi}(w^{se})$ and $\lim_{w \uparrow w^{se}} \Delta(w) = 0$.

Next, we construct w^c. For any $w \in [R^{-1}(w^{se}), w^{se}]$, define

$$G(w) \equiv \hat{U}(w - R^{-1}(w^{se})) - (U(w^{se} - R^{-1}(w) + \Delta(w)) + \Delta(w)).$$

Then, $G(\cdot)$ is strictly increasing, continuous, and strictly crosses 0 from below. Define $w^c \in (R^{-1}(w^{se}), w^{se})$ as the unique wealth such that $G(w^c) = 0$.

Suppose $w_t > w^c$ satisfies $\Pi_c(w_t) > \hat{\Pi}(w_t)$, with corresponding household-optimal choices $(c_t, p_t, _t)$. Towards contradiction, suppose $w_{t+1} > w^c$. Then, this household can exit and choose $\hat{c}_t = \hat{p}_t = p_t = c_t - f_t$ and $\hat{f}_t = 0$, where $p_t \geq 0$ because $w^c \geq R^{-1}(w^{se})$. This deviation leaves w_{t+1} unchanged and results in continuation payoff $\hat{\Pi}(w_{t+1})$.

This deviation is profitable:

$$\begin{aligned}
(1-\delta)\hat{U}(\hat{c}_t)+\delta\hat{\Pi}(w_{t+1}) &\geq (1-\delta)\hat{U}(c_t-f_t)+\delta\hat{\Pi}(w^c) \\
&> (1-\delta)(U(c_t)+\Delta(w^c))+\delta\hat{\Pi}(w^c) \\
&= (1-\delta)(U(c_t)+\Delta(w^c))+\delta\Pi^*(w^{se})-(1-\delta)\Delta(w^c) \\
&= (1-\delta)U(c_t)+\delta\Pi^*(w^{se}) \\
&\geq (1-\delta)U(c_t)+\delta\Pi^*(w_{t+1}) \\
&= \Pi^*(w_t).
\end{aligned}$$

Here, the first line follows from $\hat{c}_t = c_t - f_t$ and $w_{t+1} \geq w^c$; the second line is proven below; the third line from the definition of $\Delta(w^c)$; and the fifth line because, by Lemma 2, $w_{t+1} \leq w^{se}$. The fourth and sixth lines are algebra.

The second line follows because for any $w_t > w^c$, $\hat{U}(c_t - f_t) > U(c_t) + \Delta(w^c)$. To see this, note that

$$w^{se} \geq w_{t+1} = R(w_t - p_t) > R(w^c - p_t).$$

Hence, $p_t > w^c - R^{-1}(w^{se})$. Similarly,

$$w^c < w_{t+1} = R(w_t - p_t) \leq R(w^{se} - p_t),$$

so $p_t < w^{se} - R^{-1}(w^{se})$. Therefore,

$$\begin{aligned}
\hat{U}(c_t-f_t)=\hat{U}(p_t) &> \hat{U}(w^c - R^{-1}(w^{se})) \\
&\geq U(w^{se}-R^{-1}(w^c)+\Delta(w^c))+\Delta(w^c) \\
&> U(p_t+\Delta(w^c))+\Delta(w^c) \\
&\geq U(p_t+\Delta(w_t))+\Delta(w^c) \\
&\geq U(c_t)+\Delta(w^c).
\end{aligned}$$

Here, the first and third lines follow from $w^{se} - R^{-1}(w^c) > p_t > w^c - R^{-1}(w^{se})$; the second line from $G(w^c) = 0$; the fourth line from the fact that $w^c < w_t$ and $\Delta(\cdot)$ strictly decreasing;

and the last line from $p_t + \Delta(w_t) \geq p_t + f_t = c_t$.

The household therefore has a profitable deviation if $w_{t+1} > w^c$, so $w_{t+1} \leq w^c$ whenever $w_t \in (w^c, w^{se})$. Since $(w_t)_{t=0}^{\infty}$ is monotone by Lemma 5, it converges. Hence, $\lim_{t\to\infty} w_t \leq w^c \equiv w^{tr}$.

The final step is to show $\mathcal{W} \cap (w^c, w^{se})$ has positive measure. By definition of w^{se}, we can find $w \in (w^c, w^{se})$ such that $\Pi^*(w) > \hat{\Pi}(w)$. Since $\Pi^*(\cdot)$ is increasing and $\hat{\Pi}(\cdot)$ is continuous, $\Pi^*(\cdot) > \hat{\Pi}(\cdot)$ on an open set around w. ■

5 Policy Simulations

This section explores how policies can help left-behind communities. Most such policies are **person-based**, in the sense that they are available to households regardless of where the reside. However, policy-makers have recently proposed two other approaches that condition on the household's location. **Mobility-based** policies, such as the "Moving to Opportunity" housing program in the United States, encourage households to move to areas with greater opportunities (Katz et al. 2001, Chetty et al. 2016). **Place-based** policies, in contrast, provide business subsidies, infrastructure, and other benefits that are only accessible to those who stay in their community (Austin et al. 2018, Bartik 2020).

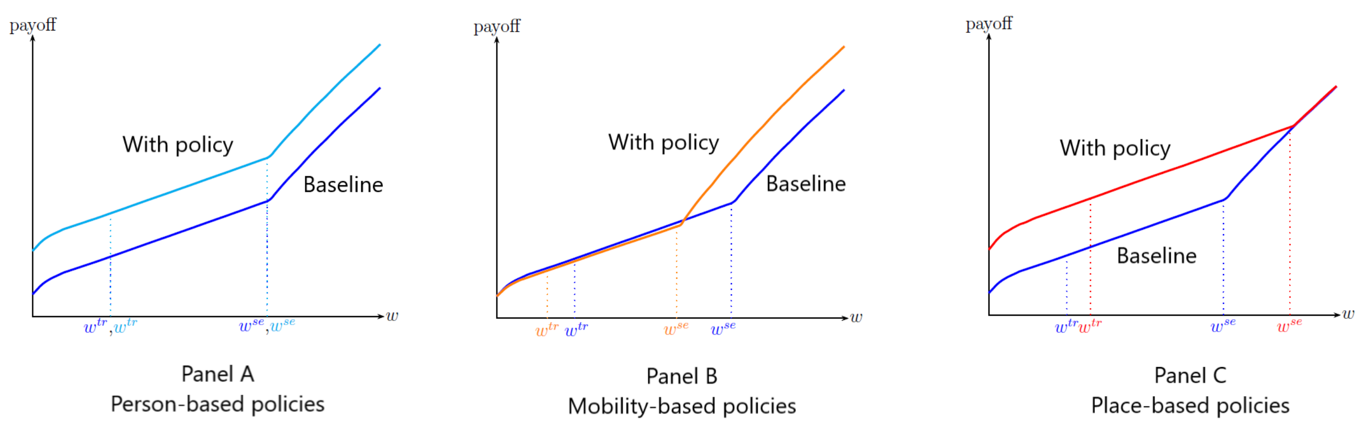

Figure 3: Simulated effects of person-, mobility-, and place-based policies.

Using numerical simulations, we illustrate that mobility- and place-based approaches have contrasting effects on wealth and welfare. We model each type of policy as increasing the household's per-period utility by a small amount. Mobility-based policies provide this per-period benefit only after the household moves to the city; place-based policies provide this benefit only while the household remains in the community; and person-based policies provide this benefit regardless of the household's location. We assume that following a deviation, the household either leaves the community, or it stays and plays the optimal equilibrium subject to the constraint $f_t = 0$.[3]

Figure 3 presents these simulations. Person-based policies do not affect the household's *relative* payoff from the community versus the city. Therefore, such policies increase the household's payoff but do not substantially influence either selection or wealth dynamics.

In contrast, both mobility- and place-based policies directly influence wealth dynamics. Mobility-based policies disproportionately increase the household's payoff from leaving the community. Such policies therefore induce households with wealth near w^{se} to leave. However, those that remain in the community face a higher outside option and so can engage in less favor exchange. To mitigate this effect, these households invest even less, exacerbating the "too big for their boots" effect and decreasing w^{tr}. Mobility-based policies therefore benefit households that leave while discouraging investment and decreasing welfare for those less-wealthy households that stay.

Place-based policies have the opposite effect. Since such policies increase the payoff from staying in the community relative to leaving, households in the community can engage in more favor exchange, mitigating the "too big for their boots" effect and increasing w^{tr}. The direct benefit of a place-based policy is therefore compounded by an equilibrium effect on investment, which suggests place-based policies can provide outsized benefits to left-behind communities.[4]

A similar multipler effect arises from benefits provided by the community itself. In our

[3] We impose this assumption because place-based policies can affect the household's punishment payoff by inducing them to stay following a deviation. This assumption gives us an upper bound on the household's punishment payoff.

[4] Austin et al. 2018 develops other justifications for implementing place-based policies.

model, the only reason for staying in the community is to engage in favor exchange. In practice, however, communities provide a sense of belonging, a platform for family, social, and religious interactions, and myriad other benefits. Like place-based policies, these benefits encourage households to remain part of the community, mitigating the "too big for their boots" effect and encouraging wealth accumulation. Consistent with this observation, some researchers have argued that tight-knit immigrant communities with limited outside options tend to build wealth relatively quickly (e.g., Portes and Sensenbrenner 1993).

6 Conclusion

Understanding the prevalence and persistence of "left behind" communities requires that we understand the social constraints faced by those experiencing poverty. This paper studies the sacrifices that households make in order to access support from their communities. We argue that those who grow their wealth risk being excluded from exchange. Thus, although the community improves the welfare of its members, it does so only at the cost of discouraging investment, dampening wealth accumulation, and deepening long-term inequality.

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A Routine Proofs

A.1 Proof of Proposition 1

Suppose that the household lives in the city. In any period t, since future vendors don't observe f_{t}, the household always chooses $f_t = 0$. Hence, vendor t accepts only if $p_t \geq c_t$. This means that $c_t \in [0, w_t]$ are the feasible consumptions, so that the household's equilibrium continuation payoff is at most $\hat{\Pi}(w_t)$ given wealth w_t.

The following equilibrium gives the household an equilibrium continuation payoff of $\hat{\Pi}(w_t)$. In period t, (i) the household proposes $(c_t, p_t) = (\hat{C}(w_t), \hat{C}(w_t))$; (ii) vendor t accepts if and only if $p_t \geq c_t$. Vendor t has no profitable deviation. This strategy attains $\hat{\Pi}$, so the household has no profitable deviation either.

Let $\{c_t^*\}_{t=0}^{\infty}$ be the consumption sequence in the equilibrium above, given initial wealth w. If $w = 0$, then $c_t^* = 0$ in all $t \geq 0$, so $\hat{\Pi}(0) = \hat{U}(0) = 0$ is the unique equilibrium payoff. If $w > 0$, then it must be true that $c_t^* > 0$ in every $t \geq 0$. Suppose otherwise. If $c_0^* > 0$, then let $\tau > 0$ be the first period in which $c_t^* = 0$. Consider the perturbation $c_{\tau-1} = c_{\tau-1}^* - \epsilon$, $c_\tau = c_\tau^* + \epsilon$ for some small $\epsilon > 0$. Since $\lim_{c \downarrow 0} \hat{U}'(c) = \infty$, this perturbation gives a strictly higher payoff, and it is feasible because $R'(\cdot) \geq 1$. If $c_0^* = 0$ instead, then let $\tau > 0$ be the first period in which $c_\tau^* > 0$. Then the perturbation $c_{\tau-1} = c_{\tau-1}^* + \epsilon$, $c_\tau = c_\tau^* - \epsilon$ again gives a strictly higher payoff for $\epsilon > 0$ small. Hence, $c_t^* > 0$ in every $t \geq 0$.

Next, we show that $\hat{\Pi}(w)$ is the household's *unique* equilibrium payoff. At $w = 0$, $\hat{\Pi}(0) = 0$, so the household's unique equilibrium payoff is indeed $\hat{\Pi}(0)$. For $w > 0$, the household can choose $(c_t, p_t) = ((1 - \epsilon_t)c_t^*, c_t^*)$ in every $t \geq 0$ for $\epsilon_t > 0$ small. Vendor t strictly prefers to accept. As $\epsilon_t \downarrow 0$ for all $t \geq 0$, the consumption sequence $\{(1 - \epsilon_t)c_t^*\}_{t=0}^{\infty}$ gives the household a payoff that converges to $\hat{\Pi}(w)$. So the household must earn at least $\hat{\Pi}(w)$ in any equilibrium.

Turning to properties of $\hat{\Pi}(\cdot)$, we claim that $\hat{\Pi}(\cdot)$ is strictly increasing. Pick $0 \leq w < \tilde{w}$. Let $\{c_t^*\}_{t=0}^{\infty}$ be the sequence associated with w. If the initial wealth is $\tilde{w}$, it is feasible to choose $c_0 = c_0^* + \tilde{w} - w$ and $c_t = c_t^*$ for $t \geq 1$. Since $\hat{U}(\cdot)$ is strictly increasing, so too is $\hat{\Pi}(\cdot)$.

It remains to show that $\hat{\Pi}(\cdot)$ is continuous for all $w > 0$. If $w > 0$, then $\hat{C}(w) > 0$. For $\tilde{w}$ sufficiently close to w, setting $c_0 = \hat{C}(w) + (\tilde{w} - w)$ and $c_t = \hat{C}(w_t)$ for $t \geq 1$ is feasible. The household's payoffs converge to $\hat{\Pi}(w)$ as $\tilde{w} \to w$ under this perturbation, which means that $\lim_{\tilde{w} \uparrow w} \hat{\Pi}(\tilde{w}) \geq \hat{\Pi}(w)$ and $\lim_{\tilde{w} \uparrow w} \hat{\Pi}(\tilde{w}) \geq \hat{\Pi}(w)$. Since $\hat{\Pi}(\cdot)$ is increasing, we conclude that $\hat{\Pi}(\cdot)$ is continuous at every $w > 0$.

We now show that $\hat{\Pi}(\cdot)$ is continuous at $w = 0$. Consider $\lim_{w \downarrow 0} \hat{\Pi}(w)$. Since $R'(\bar{w}) = \frac{1}{\delta}$, the line tangent to $R(\cdot)$ at $\bar{w}$ is $\hat{R}(w) = R(\bar{w}) + \frac{w - \bar{w}}{\delta}$. Since $R(\cdot)$ is concave, $R(w) \leq \hat{R}(w)$ for all $w \geq 0$. Therefore, $\hat{\Pi}(w)$ is bounded from above by the household's maximum payoff if we replace $R(\cdot)$ with $\hat{R}(\cdot)$. For consumption path $\{c_t\}_{t=0}^{\infty}$ to be feasible under $\hat{R}(\cdot)$, it must satisfy

$$(1-\delta)\sum_{t=0}^{\infty} \delta^t c_t \leq (1-\delta) w_0 + \delta R(\bar{w}) - \bar{w}.$$

This means that the payoff of a household with initial wealth w_0 is at most

$$\hat{U}((1-\delta) w_0 + \delta R(\bar{w}) - \bar{w}).$$

Pick any small $\epsilon > 0$. There exists $T < \infty$ and sufficiently small $w_0 > 0$ such that

$$\delta^T \hat{U}\left((1-\delta) R^T(w_0) + \delta R(\bar{w}) - \bar{w}\right) < \frac{\epsilon}{2},$$

where $R^T(w_0)$ denotes the function that applies $R(\cdot)$ T-times to w_0.

Consider a hypothetical setting that is more favorable to the household: we allow the household to both consume *and* save her wealth until period T, after which she must play the original city game. The household's payoff from this hypothetical is strictly larger than $\hat{\Pi}(w_0)$ and is bounded from above by

$$(1-\delta)\sum_{t=0}^{T-1} \delta^t (\hat{U}(R^t(w_0)) + \delta^T \hat{U}((1-\delta) R^T(w_0) + \delta R(\bar{w}) - \bar{w}).$$

As $w_0 \downarrow 0$, $R^T(w_0) \downarrow 0$, so $R^t(w_0) \downarrow 0$ for any $t < T$. Thus,

$$\hat{\Pi}(w_0) \leq (1-\delta)\sum_{t=0}^{T-1} \delta^t \hat{U}(R^t(w_0)) + \delta^T \hat{U}((1-\delta)R^T(w_0) + \delta R(\bar{w}) - \bar{w}) < \epsilon.$$

This is true for any $\epsilon > 0$, so $\lim_{w \downarrow 0} \hat{\Pi}(w) = 0$.

Finally, consider any equilibrium in the city. If $w_0 = 0$, then $w_t = 0$ in any $t \geq 0$. If $w_0 > 0$, then we have shown that $c_t > 0$ in every $t \geq 0$, so $w_t > c_t > 0$. A standard argument (see below) implies the following Euler equation:

$$\hat{U}'(c_t) = \delta R'(w_t - c_t)\hat{U}'(c_{t+1}). \tag{6}$$

Together with $R'(\cdot) \geq \frac{1}{\delta}$ and $U(\cdot)$ strictly concave, (6) implies $c_t \leq c_{t+1}$, and strictly so if $w_t < \bar{w}$.

Next, we argue that $\hat{C}(\cdot)$ is strictly increasing in w. Let $\{c_t\}_{t=0}^{\infty}$ and $\{\tilde{c}_t\}_{t=0}^{\infty}$ be the equilibrium consumption sequences for $w > 0$ and $\tilde{w} > w$, respectively. Suppose $c_0 \geq \tilde{c}_0$, and let $\tau \geq 1$ be the first period such that $c_t < \tilde{c}_t$, which must exist because $\hat{\Pi}(\cdot)$ is strictly increasing. Then, $c_{\tau-1} \geq \tilde{c}_{\tau-1}$, $w_{\tau-1} - c_{\tau-1} < \tilde{w}_{\tau-1} - \tilde{c}_{\tau-1}$, and $c_\tau < \tilde{c}_\tau$, so at least one of $(c_{\tau-1}, w_{\tau-1}, c_\tau)$ and $(\tilde{c}_{\tau-1}, \tilde{w}_{\tau-1}, \tilde{c}_\tau)$ violates (6). Hence, $\hat{C}(w)$ is strictly increasing in w. Therefore, $c_{t+1} \geq c_t$ implies $w_{t+1} \geq w_t$, with strict inequalities if $w_t \leq \bar{w}$.

Since $(w_t)_{t=0}^{\infty}$ is monotone, it converges on $\mathbb{R}_+ \cup \{\infty\}$. Suppose $\lim_{t\to\infty} w_t < \bar{w}$. Then, $R'(w_t)$ is uniformly bounded away from $\frac{1}{\delta}$. But $c_t = w_t - R^{-1}(w_{t+1})$, so $(c_t)_{t=0}^{\infty}$ converges. Hence, (6) is violated as $t \to \infty$. We conclude that $\lim_{t\to\infty} w_t \geq \bar{w}$. ■

A.2 Deriving the Euler Equation

Consider a household in the city, and let its optimal consumption and wealth sequence be $\{c_t^*, w_t^*\}_{t=0}^{\infty}$. We prove that if $w_0 > 0$, then

$$\hat{U}'(c_t^*) = \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$$

in every $t \geq 0$.

The proof of Proposition 1 says that $c_t^* > 0$, $c_{t+1}^* > 0$, and $w_t^* - c_t^* > 0$. Suppose that $\hat{U}'(c_t^*) > \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$. Then, we can perturb (c_t^*, c_{t+1}^*) to $(c_t^* + \epsilon, c_{t+1}^* - \chi(\epsilon))$, where $\chi(\epsilon)$ is chosen such that w_{t+2}^* remains the same as before the perturbation. In particular,

$$R(w_t^* - (c_t^* + \epsilon)) - (c_{t+1}^* - \chi(\epsilon)) = R(w_t^* - c_t^*) - c_{t+1}^*.$$

Hence, $\chi'(\epsilon) = R'(w_t^* - (c_t^* + \epsilon))$.

As $\epsilon \downarrow 0$, this perturbation strictly increases the household's payoff:

$$\begin{aligned}
\lim_{\epsilon \downarrow 0} \left\{ \hat{U}'(c_t^* + \epsilon) - \delta \hat{U}'(c_{t+1}^* - \chi(\epsilon))\chi'(\epsilon) \right\} &= \lim_{\epsilon \downarrow 0} \left\{ \hat{U}'(c_t^* + \epsilon) - \delta \hat{U}'(c_{t+1}^* - \chi(\epsilon)) R'(w_t^* - c_t^* - \epsilon) \right\} \\
&= \hat{U}'(c_t^*) + \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*) > 0.
\end{aligned}$$

This contradicts the fact that (c_t^*, c_{t+1}^*) is optimal. Using a similar argument, we can show that $\hat{U}'(c_t^*) < \delta R'(w_t^* - c_t^*)\hat{U}'(c_{t+1}^*)$ is not possible either. ■

A.3 Proof of Lemma 5

We break the proof of this lemma into four steps.

A.3.1 Step 1: Locally Bounding the Slope of $\Pi^*(\cdot)$ From Below

We claim that for any $w \in [0, w^{se})$, there exists $\epsilon_w > 0$ such that for any $\epsilon \in (0, \epsilon_w)$,

$$\Pi^*(w + \epsilon) - \Pi^*(w) > (1 - \delta)\epsilon.$$

First, suppose $\hat{\Pi}(w) \geq \Pi_c(w)$, and let $\{w_t, c_t\}_{t=0}^{\infty}$ be the wealth and consumption sequences if the household enters the city. Assumption 1 and the proof of Lemma 4 imply $R(w^{se} - \bar{c}) < w^{se}$, so $R(w_0 - \bar{c}) < w_0$. Proposition 1 says that $\{w_t\}_{t=0}^{\infty}$ is increasing, so $c_0 < \bar{c}$. Hence, there exists $\epsilon_w > 0$ such that $U'(c_0 + \epsilon_w) > 1$.

For any $\epsilon < \epsilon_w$, if $w_0 = w+\epsilon$, then the household can enter the city and choose $\hat{c}_0 = c_0+\epsilon$, with $\hat{c}_t = c_t$ in all $t > 0$. We can bound $\Pi^*(w+\epsilon)$ from below by the payoff from this strategy,

$$\Pi^*(w+\epsilon) \geq (1-\delta)\left(U(c_0+\epsilon) - U(c_0)\right) + \hat{\Pi}(w) > (1-\delta)\epsilon + \hat{\Pi}(w) = (1-\delta)\epsilon + \Pi^*(w).$$

We conclude that $\Pi^*(w+\epsilon) - \Pi^*(w) > (1-\delta)\epsilon$, as desired.

Now, suppose $\hat{\Pi}(w) < \Pi_c(w)$. Let $\{w_t, c_t, f_t\}_{t=0}^{\infty}$ be the wealth, consumption, and reward sequence in a household-optimal equilibrium. There exists $\tau \geq 0$ such that $f_\tau > 0$ for the first time in period τ; otherwise, the household could implement the same consumption sequence in the city. Choose $\epsilon_w > 0$ to satisfy $\epsilon_w < \delta^\tau f_\tau$.

For $\epsilon \in (0, \epsilon_w)$ and initial wealth $w_0 = w + \epsilon$, consider the perturbed strategy such that $\hat{w}_t = w_t$, $\hat{c}_t = c_t$, and $\hat{f}_t = f_t$ in every period *except* τ. In period τ, $\hat{f}_\tau = f_\tau - \frac{\epsilon}{\delta^\tau}$ and $\hat{p}_\tau = p_\tau + \chi$, where χ is chosen so that $\hat{w}_{t+1} = w_{t+1}$. Then, $\hat{c}_\tau = c_\tau + \chi - \frac{\epsilon}{\delta^\tau}$. Since the household stays in the community forever by Lemma 2, $w_t \leq w^{se}$ for all $t \leq \tau$, which together with $R(w^{se} - \bar{c}) < w^{se}$ and Assumption 1, implies $R'(w_t - p_t) > \frac{1}{\delta}$. Hence, $\chi > \frac{\epsilon}{\delta^\tau}$.

Under this perturbed strategy, (5) is satisfied in all $t < \tau$ because $f_t = 0$ in these periods; in $t = \tau$ because $\hat{f}_\tau < f_\tau$ and $\hat{w}_{\tau+1} = w_{\tau+1}$; and in $t > \tau$ because play is unchanged after τ. Moreover, $f_\tau - \frac{\epsilon}{\delta^\tau} > 0$ because $\epsilon < \epsilon_w$, and $\hat{f}_\tau + \hat{p}_\tau = \hat{c}_\tau$, so this strategy is feasible. Thus, it is an equilibrium. Consequently, $\Pi^*(w+\epsilon)$ is bounded from below by the household's payoff from this strategy,

$$\Pi^*(w+\epsilon) > (1-\delta)\delta^\tau \frac{\epsilon}{\delta^\tau} + \Pi_c(w) = (1-\delta)\epsilon + \Pi^*(w),$$

as desired.

A.3.2 Step 2: Moving from Local to Global Bound on Slope

Next, we show that for any $0 \leq w < w' < w^{se}$, $\Pi^*(w') - \Pi^*(w) > (1-\delta)(w' - w)$.

Let

$$z(w) = \sup\{w''|w < w'' \leq w^{se}, \text{ and } \forall w' \in (w, w''], \Pi^*(w') - \Pi^*(w) > (1-\delta)(w'-w)\}.$$

By Step 1, $z(w) \geq w$ exists. Moreover,

$$\Pi^*(z(w)) - \Pi^*(w) \geq \lim_{\tilde{w}\uparrow z(w)} \Pi^*(\tilde{w}) - \Pi^*(w) \geq (1-\delta)(z(w)-w),$$

where the first inequality follows because $\Pi^*(\cdot)$ is increasing, and the second inequality follows by definition of $z(w)$.

Suppose that $z(w) < w^{se}$. By Step 1, there exists $\epsilon_{z(w)}$ such that for any $\epsilon < \epsilon_{z(w)}$,

$$\Pi^*(z(w)+\epsilon) - \Pi^*(z(w)) > (1-\delta)\epsilon.$$

Hence,

$$\Pi^*(z(w)+\epsilon)-\Pi^*(w) = \Pi^*(z(w)+\epsilon)-\Pi^*(z(w))+\Pi^*(z(w))-\Pi^*(w) > (1-\delta)\epsilon+(1-\delta)(z(w)-w).$$

This contradicts the definition of $z(w)$, so $z(w) \geq w^{se}$.

For any $w' < w^{se}$, $w' < z(w)$ and so $\Pi^*(w') - \Pi^*(w) > (1-\delta)(w'-w)$, as desired.

A.3.3 Step 3: Investment is increasing in wealth.

Consider two wealth levels, $0 \leq w_L < w_H < w^{se}$, and suppose that $\Pi_c(w_L) > \hat{\Pi}(w_L)$ and $\Pi_c(w_H) > \hat{\Pi}(w_H)$. Given any household-optimal equilibria, let p_H, p_L be the respective period-0 payments under w_H, w_L. We prove that $w_H - p_H \geq w_L - p_L$. Define $I_k \equiv w_k - p_k$, $k \in \{L, H\}$. Towards contradiction, suppose that $I_H < I_L$.

We first show that $c_H > c_L + (w_H - w_L)$. Suppose instead that $c_H \leq c_L + (w_H - w_L)$.

Since $I_H < I_L$, we have $p_H > p_L + (w_H - w_L)$. But then $f_H < f_L$, since

$$f_H = c_H - p_H < c_H - (p_L + (w_H - w_L)) \leq c_L + (w_H - w_L) - (p_L + (w_H - w_L)) = f_L.$$

Consider the following perturbation: $\hat{p}_H = p_L + (w_H - w_L) \in (p_L, p_H)$, $\hat{f}_H = f_H + p_H - \hat{p}_H \geq f_H$, and $\hat{c}_H = c_H$. Under this perturbation, $\hat{I}_H = w_H - \hat{p}_H = I_L$. Thus, to show that the perturbation satisfies (5), we need only show that $\hat{f}_H \leq f_L$. Indeed:

$$\hat{f}_H = f_H + p_H - (p_L + (w_H - w_L)) = c_H - (c_L - f_L) - (w_H - w_L) = f_L + c_H - (c_L + w_H - w_L) \leq f_L,$$

where the final inequality holds because $c_H \leq c_L + (w_H - w_L)$ by assumption. Thus, this perturbation is also an equilibrium.

We claim that a household with initial wealth w_H strictly prefers this equilibrium to the original equilibrium, which is true so long as

$$\begin{aligned} & (1-\delta)(U(c_H) - \hat{f}_H) + \delta\Pi^*(R(I_L)) > (1-\delta)(U(c_H) - f_H) + \delta\Pi^*(R(I_H)) \\ \iff & (1-\delta)(\hat{f}_H - f_H) < \delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))) \\ \iff & (1-\delta)(p_H - \hat{p}_H) < \delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))). \end{aligned}$$

We know that $I_L = I_H + p_H - \hat{p}_H$. Since the household stays in the community, $I_H < I_L < w^{se}$, so $R'(I_H), R'(I_L) > \frac{1}{\delta}$. Thus,

$$R(I_L) - R(I_H) > \frac{1}{\delta}(I_L - I_H) = \frac{1}{\delta}(p_H - \hat{p}_H).$$

By Step 2, $\Pi^*(\cdot)$ increases at rate strictly greater than $(1-\delta)$, so we conclude

$$\delta(\Pi^*(R(I_L)) - \Pi^*(R(I_H))) > \delta(1-\delta)\frac{1}{\delta}(p_H - \hat{p}_H),$$

as desired. Thus, if $I_H < I_L$, then $c_H > c_L + (w_H - w_L)$.

We are now ready to prove that $I_H < I_L$ contradicts household optimality. To do so, we consider two perturbations: one at w_L and one at w_H. At w_H, consider setting

$$\hat{c}_H = c_L + (w_H - w_L) > c_L \geq 0,$$
$$\hat{p}_H = p_L + w_H - w_L \in (p_L, w_H],$$
$$\hat{f}_H = \hat{c}_H - \hat{p}_H = f_L.$$

By construction, $w_H - \hat{p}_H = I_L$. Thus, $\hat{f}_H$ satisfies (5) because f_L does. Moreover, $\hat{p}_H + \hat{f}_H = \hat{c}_H$, so the neighbor is willing to accept. This perturbed strategy is therefore an equilibrium. For the original equilibrium to be household-optimal, we must therefore have

$$(1-\delta)(U(c_H) - f_H) + \delta\Pi^*(R(I_H)) \geq (1-\delta)(U(\hat{c}_H) - \hat{f}_H) + \delta\Pi^*(R(\hat{I}_H)). \tag{7}$$

At w_L, consider setting

$$\hat{c}_L = c_H - (w_H - w_L) > c_L \geq 0,$$
$$\hat{p}_L = p_H - (w_H - w_L) \in (p_L, w_L] \ ,$$
$$\hat{f}_L = \hat{c}_L - \hat{p}_L = f_H.$$

By construction, $w_L - \hat{p}_L = I_H$. Thus, $\hat{f}_L$ satisfies (5) because f_H does. This perturbed strategy is again an equilibrium, so the original equilibrium is household-optimal only if

$$(1-\delta)(U(c_L) - f_L) + \delta\Pi^*(R(I_L)) \geq (1-\delta)(U(\hat{c}_L) - \hat{f}_L) + \delta\Pi^*(R(\hat{I}_L)). \tag{8}$$

Combining (7) and (8) and plugging in definitions, we have

$$U(c_H) - U(c_H - (w_H - w_L)) \geq U(c_L + (w_H - w_L)) - U(c_L).$$

However, $c_H > c_L + w_H - w_L$ and $U(\cdot)$ is strictly concave, so this inequality cannot hold. Thus, if $I_H < I_L$, then at least one of the equilibria at w_H and w_L cannot be household-optimal.

A.3.4 Step 4: Establishing Monotonicity

We have shown that investment, $I(w)$, is increasing in w. Consider a household-optimal equilibrium with $w_1 \geq w_0$. Then, $I(w_1) \geq I(w_0)$, so $w_2 = R(I(w_1)) \geq R(I(w_0)) = w_1$. Thus, $w_2 \geq w_1$, and $w_{t+1} \geq w_t$ for all $t > 1$ by the same argument. Similarly, if $w_1 \leq w_0$, then $I(w_1) \leq I(w_0)$, $w_2 \leq w_1$, and $w_{t+1} \leq w_t$ in all $t \geq 0$. We conclude that $(w_t)_{t=0}^{\infty}$ is monotone in any household-optimal equilibrium. ■

B Reversible Exit

B.1 Household can return to the community

This appendix shows that mis-investment occurs even if the household can return to the community after leaving for the city. Formally, we modify the game in Section 2 so that at the start of every period while the household is in the city, it can return to the community. If it does, then it plays the community game until it again chooses to leave for the city. Payoffs and information structures are the same as in Section 2, and so neighbors observe all of the household's interactions with neighbors, while vendors observe only their own interactions.

We impose a slightly stronger version of Assumption 1.

Assumption 2 *Define $\bar{c}_m$ as the solution to $\hat{U}'(\bar{c}_m) = 1$, and let $\hat{w}_m$ satisfy $R(\hat{w}_m - \bar{c}_m) = \hat{w}_m$. Then, $R'(\hat{w}_m) > \frac{1}{\delta}$.*

Under this assumption, we can prove that mis-investment occurs even if exit is reversible.

Proposition 3 *Impose Assumption 2. There exists a w^{**}, a $w^{cc} < w^{**}$, and a positive-measure interval $\mathcal{W} \subseteq [0, w^{**})$ such that the household permanently exits the community if $w_0 \notin \mathcal{W}$. If $w_0 \in \mathcal{W}$, then in any household-optimal equilibrium, the household is in the community for an infinite number of periods.*

Moreover, if $w_0 \notin \mathcal{W}$, then in any equilibrium,

$$\lim_{t\to\infty} w_t > w^{**},$$

while if $w_0 \in \mathcal{W}$, then for every $t \geq 0$, $w_t < w^{**}$. Moreover, $w_{t+1} < w_t$ whenever $w_t \in (w^{cc}, w^{**})$.

B.1.1 Proof of Proposition 3

Much like the proof of Proposition 2, we break this proof into a sequence of lemmas. We begin by showing that the household's worst equilibrium payoff equals $\hat{\Pi}(\cdot)$, its worst equilibrium payoff from the game with reversible exit.

Lemma 6 *For any initial wealth $w \geq 0$, the household's worst equilibrium payoff is $\hat{\Pi}(\cdot)$.*

Proof of Lemma 6: This proof is similar to the proof of Proposition 1. It is an equilibrium for $f_t = 0$ in every $t \geq 0$, in which case it is optimal for the household to permanently leave the community. In the city, vendor t accepts only if $p_t \geq c_t$. Therefore, $\hat{\Pi}(\cdot)$ gives the maximum equilibrium payoff if the household permanently leaves the community. But as in the proof of Proposition 1, the household cannot earn less than $\hat{\Pi}(\cdot)$, because vendor t must accept whenever $p_t > c_t$. $\square$

Now, we turn to the household's maximum equilibrium payoff. Define $\Pi^{**}(w)$ as the maximum equilibrium payoff with initial wealth w. Define $\Pi_c^*(\cdot)$ identically to $\Pi_c(\cdot)$, except that $\Pi^*(\cdot)$ is replaced by $\Pi^{**}(\cdot)$. Define

$$\Pi_m^*(w) \equiv \max_{0\leq c\leq w} \left\{(1-\delta)\hat{U}(c) + \delta\Pi^{**}(R(w-c))\right\}$$

as the household's maximum equilibrium payoff if it chooses the city in the current period. The key difference between this model and the baseline model is that $\Pi_m^*(\cdot)$ might entail the household returning to the community to take advantage of relational contracts in the future. Therefore, $\Pi^*_m(\cdot) \geq \hat{\Pi}(\cdot)$, since the latter entails staying in the city forever.

Lemma 7 *Both* $\Pi^*_c(\cdot)$ *and* $\Pi^*_m(\cdot)$ *are strictly increasing. For all* $w \geq 0$, $\Pi^{**}(w) \equiv \max\{\Pi^*_c(w), \Pi^*_m(w)\}$.

Proof of Lemma 7: By Lemma 6, the household earns no less than $\hat{\Pi}(\cdot)$ following a deviation. As in Lemma 1, conditional on choosing the community in period 0, the household's maximum equilibrium payoff equals $\Pi^*_c(w_0)$. If the household instead chooses the city in period t, then $f_t = 0$ in any equilibrium., since the continuation equilibrium is independent of f_t. Thus, the household optimally sets $p_t = c_t$, so its maximum equilibrium continuation payoff equals $\Pi^{**}(R(w-c))$. We conclude that $\Pi^*_m(w)$ is the household's maximum equilibrium payoff conditional on choosing the city. It then immediately follows that $\Pi^{**}(w) \equiv \max\{\Pi^*_c(w), \Pi^*_m(w)\}$. Both $\Pi^*_c(\cdot)$ and $\Pi^*_m(\cdot)$ are strictly increasing by inspection. $\square$

Apart from some details of the proof, the next result is similar to Lemma 2.

Lemma 8 *If* $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$, *then* $\Pi^{**}(w_t) > \hat{\Pi}(w_t)$ *in all* $t \geq 0$ *of any household-optimal equilibrium.*

Proof of Lemma 8: Suppose not, and let $\tau > 0$ be the first period such that $\Pi^{**}(w_\tau) = \hat{\Pi}(w_\tau)$. In period $\tau - 1$, (5) implies that $f_t = 0$ if the household stays in the community. Therefore, it is optimal for the household to leave the community in $\tau - 1$. But then it is optimal for the household to *permanently* leave the community in $\tau - 1$, since $\Pi^{**}(w_\tau) = \hat{\Pi}(w_\tau)$. So $\Pi^{**}(w_{\tau-1}) = \hat{\Pi}(w_{\tau-1})$, contradicting the definition of τ. $\square$

Our next result is the analogue to Lemma 3 in this setting.

Lemma 9 *Suppose that* $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$. *Then in every* $t \geq 0$, $\hat{U}'(c_t) \geq 1$, *and there exists* $\tau > t$ *such that the household stays in the community in period* τ.

Proof of Lemma 9: Towards contradiction, suppose that there exists $t \geq 0$ such that $\hat{U}'(c_t) < 1$, so that *a fortiori*, $U'(c_t) < 1$. If $f_t > 0$, then we can decrease f_t and c_t by the same $\epsilon > 0$. This perturbation is also an equilibrium, and increases the household's period-t payoff at rate $1 - \hat{U}'(c_t) > 0$ as $\epsilon \to 0$. Thus, $f_t = 0$, which implies that $p_t = c_t > 0$.

By Lemma 8, $\Pi^*(w_{t+1}) > \hat{\Pi}(w_{t+1})$. Therefore, there exists a $\tau > t$ such that $f_\tau > 0$, since otherwise the household could do no better than exiting the city permanently. Let τ be the *first* period after t such that $f_\tau > 0$. Note that the household must be in the community in period τ.

Consider the following perturbation: decrease p_t and c_t by $\epsilon > 0$, and increase p_τ and decrease f_τ by $\chi(\epsilon)$, where $\chi(\epsilon)$ is chosen so that $w_{\tau+1}$ remains constant. Then, $\chi(\epsilon) \geq \frac{\epsilon}{\delta^{\tau-t}}$ because $R'(\cdot) \geq \frac{1}{\delta}$. As $\epsilon \to 0$, $\chi(\epsilon) \to 0$. Hence, this perturbation is feasible for small enough $\epsilon > 0$. It is an equilibrium, since (5) is trivially satisfied in all $t' \in [t, \tau - 1]$ because $f_{t'} = 0$ in those periods. This perturbation changes the household's period-t continuation payoff at rate no less than

$$-(1-\delta)\hat{U}'(c_t) + \delta^{\tau-t}(1-\delta)\frac{1}{\delta^{\tau-t}} > 0$$

as $\epsilon \to 0$. Thus, the original equilibrium could not have been household-optimal. □

Next, we show that the household stays in the community for sufficiently low initial wealth levels.

Lemma 10 *The set*

$$\left\{w | \Pi^{**}(w) > \hat{\Pi}(w)\right\}$$

has positive measure, with

$$w^{**} \equiv \sup\left\{w | \Pi^{**}(w) > \hat{\Pi}(w)\right\} < \infty.$$

Proof of Lemma 10: The proof that $\left\{w | \Pi^{**}(w) > \hat{\Pi}(w)\right\}$ is identical to the proof in Lemma 4, since the same constructions work at $w = 0$, $\Pi^{**}(\cdot)$ is increasing, and $\hat{\Pi}(\cdot)$ is continuous. To prove that $w^{**} < \infty$, suppose w_0 is such that

$$R(w_0 - \bar{c}_m) > w_0.$$

For any such w_0,

$$\hat{\Pi}(w_0) > \hat{U}(\bar{c}_m) \geq \Pi^{**}(w_0),$$

where the second inequality follows by Lemma 9 and the assumption that $\Pi^{**}(w) > \hat{\Pi}(w)$. Contradiction. So $\hat{\Pi}(w_0) = \Pi^{**}(w_0)$ for any $w_0 > \hat{w}_m$. We conclude $w^{**} < \infty$. $\square$

We are now in a position to prove Proposition 3. So far, the argument has hewn closely to the proof of Proposition 2. The rest of the proof marks a more substantial departure.

Lemma 10 shows that a positive-measure set $\mathcal{W}$ exists such that $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$ for all $w_0 \in \mathcal{W}$. Lemma 8 implies that for any $w_0 \in \mathcal{W}$, we can construct an infinite sequence of periods such that the household remains in the community for each period in that sequence. For any $w_0 \notin \mathcal{W}$, $\Pi^{**}(w_0) = \hat{\Pi}(w_0)$ and so the household permanently exits the community. This proves the first part of Proposition 3.

From the proof of Lemma 9, we know that $w^{**} \leq \hat{w}_m$. Assumption 2 then implies that $R'(w^{**}) > \frac{1}{\delta}$. As in the proof of Proposition 2, if the household permanently exits the community, then w_t is increasing, with $\lim_{t\to\infty} w_t > w^{**}$. This proves the second part of Proposition 3.

Suppose $w_0 \in \mathcal{W}$. Then, Lemma 8 and the definition of w^{**} immediately imply that $w_t < w^{**}$ in every $t \geq 0$. It remains to identify a $w^{cc} < w^{**}$ such that if $w_0 \in \mathcal{W}$, then whenever $w_t \in (w^{cc}, w^{**})$ in a household-optimal equilibrium, $w_{t+1} < w_t$. By an argument similar to Proposition 2,

$$\lim_{w \uparrow w^{**}} \Pi^{**}(w) = \Pi^{**}(w^{**}) = \hat{\Pi}(w^{**}),$$

because $\Pi^{**}(\cdot)$ is increasing, $\hat{\Pi}(\cdot)$ is continuous, $\Pi^{**}(\cdot) \geq \hat{\Pi}(\cdot)$, and $\Pi^{**}(w) = \hat{\Pi}(w)$ for all

$w > w^{**}$. Therefore, we can define a wealth level $w^{cc1} < w^{**}$ similarly to w^c in the proof of Proposition 2.

Define the function

$$F(w) \equiv (1-\delta)\hat{U}(w^{**} - R^{-1}(w)) + \delta\hat{\Pi}(w^{**}) - \hat{\Pi}(w)$$

on $w \in [0, w^{**}]$. Then $F(\cdot)$ is strictly decreasing and continuous, with

$$F(0) = (1-\delta)\hat{U}(w^{**}) + \delta\hat{\Pi}(w^{**}) > 0.$$

At $w_0 = w^{**}$, it is feasible for the household to permanently leave the community and consume $c_t = w^{**} - R^{-1}(w^{**})$ in each $t \geq 0$. However, doing so violates (6), since $R'(w^{**}) > \frac{1}{\delta}$. Therefore, this consumption path must be dominated by some other feasible consumption path once the household permanently leaves the community, which implies

$$\hat{U}(w^{**} - R^{-1}(w^{**}) < \hat{\Pi}(w^{**}).$$

Consequently,

$$F(w^{**}) = (1-\delta)\hat{U}(w^{**} - R^{-1}(w^{**})) - (1-\delta)\hat{\Pi}(w^{**}) < 0.$$

We conclude that there exists a unique $w^{cc2} \in (0, w^{**})$ such that $F(w^{cc2}) = 0$.

Set $w^{cc} = \max\{w^{cc1}, w^{cc2}\}$. For any $w_0 \in (w^{cc}, w^{**})$ such that $\Pi^{**}(w_0) > \hat{\Pi}(w_0)$, the household must either stay in the community or leave in $t = 0$. If it stays in the community, then we can follow the steps of the proof of Proposition 2, Statement 3, to conclude that $w_{t+1} \leq w^{cc1} < w_t$.

Suppose the household is in the city in $t = 0$. Towards contradiction, suppose that

$w_{t+1} \geq w_t$. Therefore, the household's payoff satisfies

$$\begin{aligned}
\Pi^{**}(w_0) &\leq (1-\delta)\hat{U}(w^{**} - R^{-1}(w_0)) + \delta\Pi^{**}(w^{**}) \\
&= (1-\delta)\hat{U}(w^{**} - R^{-1}(w_0)) + \delta\hat{\Pi}(w^{**}) \\
&< \hat{\Pi}(w_0),
\end{aligned}$$

where the first inequality follows because $f_0 = 0$, so that $p_0 = c_0 = w_0 - R^{-1}(w_1) \leq w^{**} - R^{-1}(w_0)$; the equality follows because $\Pi^{**}(w^{**}) = \hat{\Pi}(w^{**})$, and the final, strict inequality follows because $w_0 > w^{cc2}$ and so $F(w_0) < 0$. But $\Pi^{**}(\cdot) \geq \hat{\Pi}(\cdot)$, proving a contradiction. So $w_{t+1} < w_t$ if the household is in the city in $t = 0$.

We conclude that if $w_0 \in \mathcal{W}$, then in any household-optimal equilibrium, $w_t < w^{**}$ for all $t \geq 0$, and $w_{t+1} < w_t$ whenever $w_t > w^{cc}$. This completes the proof. ■