MANAGERIAL ECONOMICS

LECTURE 8: GAME THEORY

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Aims of this lecture

- We will discuss a popular method to analyze strategic behavior
- Game theory is a useful tool to analyze competitive situations

You should read Chapter 12 in Allen et al., *Managerial Economics* (8th ed.).

- Further reading: Martin J. Osborne, 2004, An Introduction to Game Theory, Oxford University Press
- Master program: Game Theory

Why “game” theory?

Historic reasons, games were the first situations that were analyzed.

Modern treatment goes back to John von Neuman and Oskar Morgenstern.

Because of the (possible) interactions (remember oligopolists!), decision makers (“players”) try to anticipate the actions of the other players and to decide on a plan action (“form a strategy”).

Jargon

Game theorists use a lot of jargon...

A game

"is a description of strategic interaction that includes the constraints on the actions that the players can take and the players' interests, but does not specify the actions that the players do take"
(Osborne and Rubinstein, A course in game theory, p2)

In other words, we have a situation where actors, with limited choices, interact to pursue their goals. Actors may have perfect or imperfect information, and they may behave cooperatively or uncooperatively.

Players and Strategies

A player

is the basic unit that makes a decision in a game.

A strategy

is a plan of action.

Often, it is assumed that the strategy is decided once for all possible outcomes of events (and the player does not deviate from this strategy).

Other important parts

- Feasible strategy: A plan of decisions that are allowed by the rules of the game.
- Payoff (matrix): The outcomes depending on the chosen strategies for each player.
- Order: Who decides when? This can be simultaneous or sequential.

Assumptions

The most simple game theoretical models use the following assumptions:

- Rationality: Players are rational, i.e., they know what they are doing (know how their actions influence others), form expectations if information is incomplete, knows what s/he wants, and, after some thought ("optimization"), forms a strategy.
- This assumption builds on typical assumptions used in micro-economics: well-behaved preferences and utility functions.
- In more complex models, this assumption on rational behavior can be relaxed, e.g., by assuming "bounded rationality".
- Assumptions on preferences are flexible enough to allow for e.g., altruistic behavior.

Uncertainty

Players often have to make decisions when they have incomplete information:

- Environment: Players might not know the objective state of the environment
- Game: Players might not know the rules of the game
- Other players: Players might have incomplete knowledge how others might behave
- Chance: Players might not know how random events influence their actions

In all these situations, it is assumed that players can form expectations. (This is often contested, e.g., by psychologists who show in experiments that humans are quite bad when it comes to forming expectations.)

Describing a game

Games are often described by

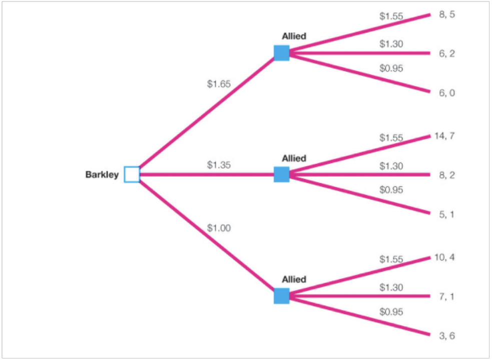
- Matrix form: A table that summarizes all possible outcomes
- Extensive form: A graph that maps the players, theirs decisions, and the outcomes. Aka “game trees”.
- Algebra: A set of mathematical formulae that define the game, the players, their actions, preferences, et cet.

A matrix representation of a game

		Barkley's strategy	
		Spend at current level	Increase spending
Allied's strategy	Spend at current level	3, 4	2, 3
	Increase spending	4, 3	3, 2

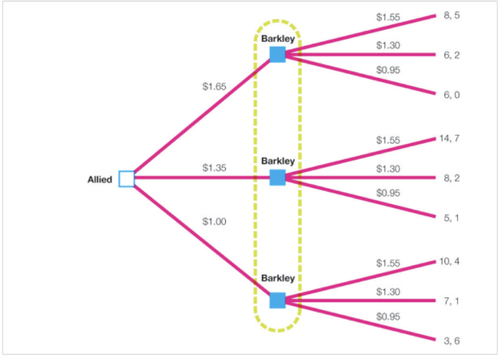
Notes: There are two players, A and B, who move simmultaneously. Each player has the same actions, keep the status quo or increase spending. The resulting outcomes are indicated in the cells where the first number is A's reward and the second number is B's reward. Figure 11.1 in Allen et al.,

A game tree representation of a game



Notes: There are two players, A and B, who move sequentially. B moves first and chooses between 1, 1.35, and 1.65. After B made the decision, A decides. In this game, A has the same choices, independent of B's initial action, and decides between 0.95, 1.3, and 1.55. The numbers on the right describe the outcomes, where the first number is the first mover's payoff (B) and the second is the second mover's payoff (A). Figure 12.2 in Allen et al., *Managerial Economics* (8th ed.), p465.

A game tree representation of a simultaneous game



Notes: There are two players, A and B, who move simultaneously. A chooses between 1, 1.35, and 1.65. B decides *at the same time* and decides between 0.95, 1.3, and 1.55. The numbers on the right describe the outcomes, where the first number is the first player's payoff (A) and the second is the second Player's payoff (B). The dotted line indicates that A and B decide at the same time. Figure 11.3 in Allen et al., *Managerial Economics* (7th ed.). (Note this differs from Fig. 12.3 in the 8th edition where A and B are exchanged.)

Representation is a matter of taste

		Allied's pricing strategies		
		$0.95	$1.30	$1.55
Barkley's pricing strategies	$1.00	3, 6	7, 1	10, 4
	$1.35	5, 1	8, 2	14, 7
	$1.65	6, 0	6, 2	8, 5

Notes: This is the matrix representation of the game on slide 11 above. Figure 12.4 in Allen et al., *Managerial Economics* (8th ed.), p471. (Note the book states this is the representation of the sequential game of slide 10. This is, strictly speaking not true, as the matrix notation assumes simultaneous decisions.)

How do we find out what is going to happen?

The key concept is the “ equilibrium”:

- Equilibrium: No player has an incentive to unilaterally change his or her strategy
- No player is able to improve his or her payoff by unilaterally changing strategy.

In other words, players decide on their strategies and accept the associated payoff.

Backward induction

Backward induction describes the process with which we may determine the strategies and outcomes:

- Rank the possible outcomes
- Identify the decisions that lead to these outcomes
- Will the action of the other players lead to this outcome? If not, choose another action or analyze the next-best outcome.
- et cetera

Example, sequential game on slide 10 above:

- B prefers (14,7) to all other outcomes (14 is more than what can be obtained in all other sceenarios.)
- B would want to choose 1.35.
- Given B chose 1.35, A prefers (14,7) to (8,2) and (5,1).
- B: 1.35; A: 1.55; (14.7) *dominates* all other choices.

Dominant strategies

Dominant strategy

A strategy whose payout in any outcome is greater relative to all other feasible strategies

- A strategy that is optimal regardless of the strategies selected by rivals
- Basically, any strategic element is taken out of the game (simplest possible form of a game)

Example, simultaneous game on slide 9 above:

- Independent of B's actions, A's outcomes are always better if A chooses to increase spending: (4,3) $>$ (3,4) and (3,2) $>$ (2,3).
- Independent of A's actions, B's outcomes are always better if B chooses to maintain current spending: (3,4) $>$ (2,3) and (4,3) $>$ (3,2).

Dominated strategies

Dominated strategy

A strategy which is always dominated by other strategies.

- A dominated strategy will never be chosen
- We might solve for the equilibrium by sequentially removing dominated strategies from the set of choices

Example, simultaneous game on slide 11 above:

- B's strategy of \$1.00 is always inferior to choosing \$1.35: (3,6) $<$ (5,1) and (7,1) $<$ (8,2) and (10,4) $<$ (14,7)
- A's strategy of \$1.30 is always inferior to choosing \$1.55: (7,1) $<$ (10,4) and (8,2) $<$ (14,7) and (6,2) $<$ (8,5)

Elimination of dominated strategies

A. Barkley's $1.00 strategy is eliminated.

Barkley's pricing strategies \ Allied's pricing strategies	$0.95	$1.30	$1.55
$1.35	5, 1	8, 2	14, 7
$1.65	6, 0	6, 2	8, 5

B. Allied's $0.95 and $1.30 strategies are eliminated.

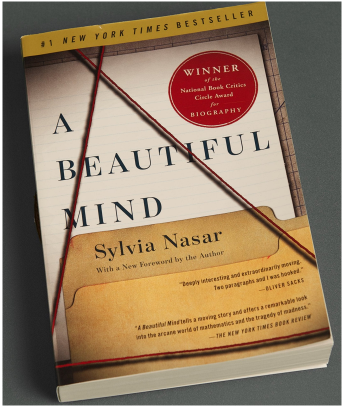
Barkley's pricing strategies \ Allied's pricing strategies	$1.55
$1.35	14, 7
$1.65	8, 5

Notes: Iteratively eliminating dominated strategies allows to analyze a simpler game. In this example, A always chooses 1.55. Since B has a greater payoff from choosing 1.35 than 1.65 (14 $<$ 8), the equilibrium outcome can be described by the set of choices (1.35, 1.55) resulting in the payoff (14,7).

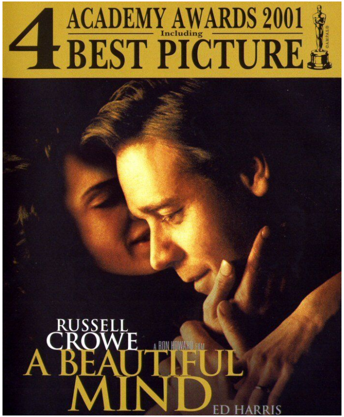
The Nash equilibrium

- John Nash (Nobel Prize 1994) formulated *one* definition of an equilibrium
  - http://nobelprize.org/economics/laureates/1994/nash-autobio.html
  - http://www.abeautifulmind.com
- Nash equilibrium: Assuming that all players are rational, every player should choose the best strategy conditional on all other players doing the same
- If no player has an incentive to deviate from a chosen strategy, then the strategy is a Nash-Equilibrium
- Note: all equilibria in dominant strategies are automatically also Nash-equilibria
- Note: A Nash equilibrium might not exist or there could be several Nash equilibria!
- Note: There are other concepts of equilibria, the Nash-equilibrium is powerful, widely used, but not the only thing under the sun!

Sequential moves



First, the book ...



Then, the movie ...

What shall we do?

			Allied	
		Product alpha	Product beta	Product zeta
Barkley	Product lambda	4, 6	9, 8	6, 10
	Product pi	6, 8	8, 9	7, 8
	Product sigma	9, 8	7, 7	5, 5

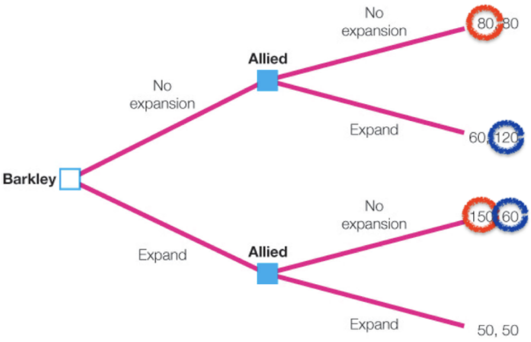
Notes: Two firms consider the introduction of new products. The new products affect profits differently, depending on whether they are complements or substitutes. The two firms do not know which product their competitor chooses, but they do know what the effects will be. Which products will they introduce?Figure 12.6 in Allen et al., *Managerial Economics* (8th ed.), p474.

The new products are ...

1. Delete dominated strategies: I this case, there is no dominated strategy!
2. For each choice of A (B), indicate B's (A's) best response.
   E.g., if A chooses α, B's best action is σ. Mark the number 9 somehow. (I use a circle around 9, e.g., ⑨.)
3. All cells where two numbers are circled are a Nash-equilibrium.

What's the outcome?

Finding the equilibrium with backward induction



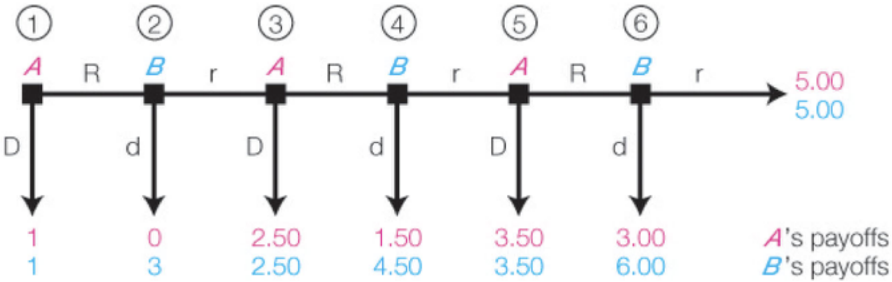
Notes: Solving through backward induction is similar to finding the equilibrium in a matrix representation. For each branch, identify the preferred outcome (here, I used circles). Each cell with a circle is a Nash-equilibrium. Figure 12.10 in Allen et al., *Managerial Economics* (8th ed.), p479.

The Centipede game

- Two players A and B play sequentially.
- Player A moves first and decides on whether to stop the game (“down”) or to continue (“right”).
- If the game continues, player B moves and decides on whether to stop the game or to continue.
- If not stopped earlier, the game ends after the 6th move (player B’s third choice).
- The payoff of the different stages are given in the Figure on the next slide.

What will happen?

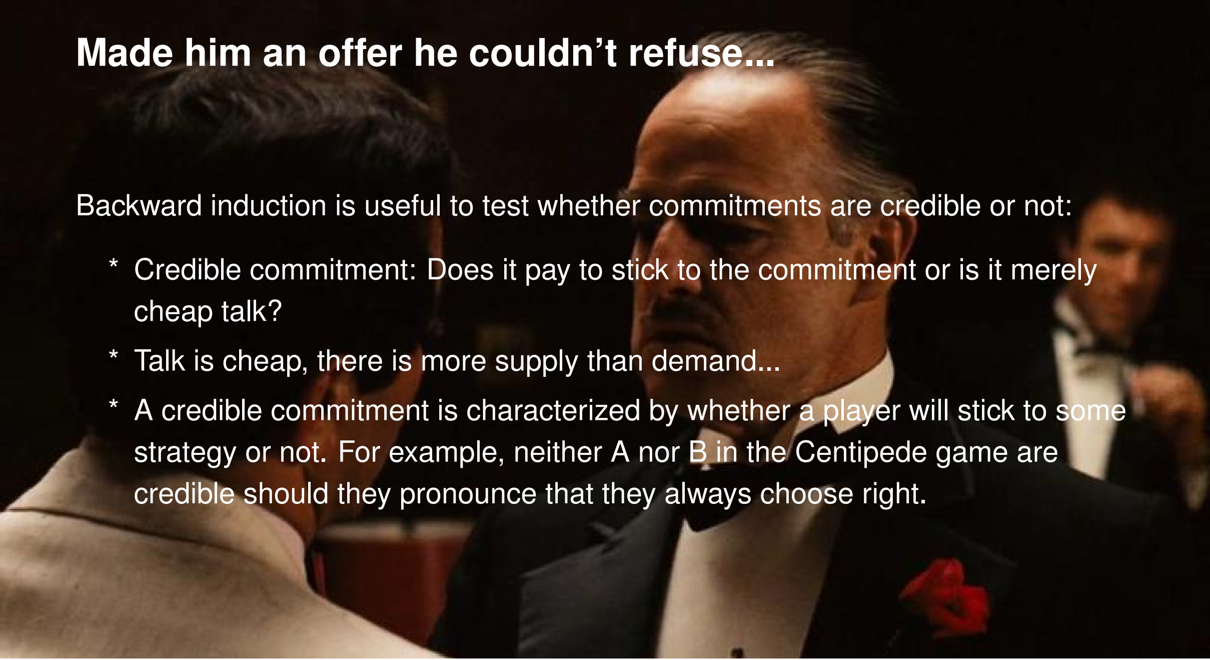
Game tree representation of the finite Centipede game



Notes: Solve this using backward induction! Figure 12.11 in Allen et al., *Managerial Economics* (8th ed.), p480.

Strategic foresight: the use of backward induction

- Backward induction: We start with the final decision. This is B's choice and the outcomes are either (3,6) if B decides to stop or (5,5) if B chooses "right". B is better off choosing to stop.
- In the pen-ultimate step, it is A's decision to stop and receive 3.5 or to continue, knowing that B will stop, and receive 3. A is better off stopping and getting 3.50.
- In round 4, B realizes that A will choose to stop, resulting in a payoff of 3.50. B is better off stopping and getting 4.50.
- et cetera
- In the first round, A realizes that B will stop in the next round, which will result in a payout of 0 and therefore chooses to stop, resulting in a payoff of (1,1).
- Note that (1,1) is much less than (5,5). However, (5,5) cannot be realized without additional institutions (e.g., enforceable contracts, threat, violence, ...



Made him an offer he couldn't refuse...

Backward induction is useful to test whether commitments are credible or not:

* Credible commitment: Does it pay to stick to the commitment or is it merely cheap talk?
* Talk is cheap, there is more supply than demand...
* A credible commitment is characterized by whether a player will stick to some strategy or not. For example, neither A nor B in the Centipede game are credible should they pronounce that they always choose right.

Ultimatum Game

- Two players decide how to divide money that is given to them, M.
- The first player proposes how to divide the sum between the two players
- The second player either accepts or rejects this proposal
  - If the second player rejects, neither player receives anything, (0,0).
  - If the second player accepts, the money is split according to the proposal, $(M - x, x)$.
- The game is played only once so that reciprocity is not an issue.
- What is optimal for the first player? What will the second player do?

Equilibrium in the Ultimatum game

- Player one offers smallest possible amount, e.g., €0.01, knowing that second player is better off accepting the offer than rejecting it, $0.01 > 0$.
- Neither player has an incentive to deviate from this!

Experimental evidence (John List, 2007):

- In real life: most people offer about 40-60%
- Offers below 20% are usually rejected

The Prisoner's dilemma

- Two prisoners are interrogated by the police. Both are accused of a crime, but it is known that only one committed the crime, while the other was merely an accomplice.
- Each prisoner has a choice, confess or do not confess.
- The associated prison sentences are given in the following table. (NB: a greater number is a worse outcome for the prisoner!)

	B	
A	Confess	Do not confess
Confess	(8,8)	(2,10)
Do not confess	(10,2)	(4,4)

The Prisoners’ Dilemma

- Perhaps the most famous game.
- Punch line: Two players who cannot coordinate their actions choose actions which are optimal from an individual perspective. If they could coordinate, their payoffs would be greater.
- Here, the social optimum is also not achieved (failure to detect severe crime, indiscriminating punishment, spending too many resources (16 instead of 12 years).
- This outcome is often taken as “proof” that economics with the assumption of rational utility maximizing agents is based on problematic assumptions.
- Even if the players could agree ex ante on (do not confess, do not confess), this is not credible as the incentive to deviate is very strong.
- However, watch this clip from a British TV show.

Setting prices as a prisoners' dilemma

- We may use game theory to analyze firms' pricing decisions.
- The interaction between rival firms often resembles a prisoners' dilemma.
- See next slide for an example.
- Here, in contrast to the framing as a social problem (punishment of innocent prisoners), it re-enforces the effect of competition on the allocation of resources.
- Since collusion and high prices are not credible, competition forces the players to price their products at a lower price. Society is better off as overall welfare increases.

A prisoners' dilemma

		Allied's pricing strategies	
		Price high	Price low
Barkley's pricing strategies	Price high	5, 5	1, 20
	Price low	20, 1	3, 3

Notes: Both players have the same strategies ("symmetric game"). If both choose to increase prices, they would achieve $(5, 5)$. However, (high, high) is not an equilibrium, either player is better off deviating. (low, low) is a stable Nash-equilibrium. Figure 12.13 in Allen et al., *Managerial Economics* (8th ed.), p484.

Meeting more than just once

Repeated games can lead to cooperative behavior:

- Trust, reputation, promises, threats, and reciprocity are relevant only if games are played often.
- Cooperative behavior is more likely if there is an infinite time horizon than if there is a finite time horizon.
- If there is a finite time horizon, then the value of cooperation, and hence its likelihood, diminishes as the time horizon is approached. Backward induction implies that cooperation will not take place in this case.

Folk Theorem

Any type of behavior can be supported by an equilibrium when the players have sufficiently high expectations that they will interact again.

Credible threats and promises (may) matter!

Repeated games: Cartel

Assume two firms want to jointly set a high price to maximize profits in the cartel.

- Each firm has an incentive to deviate and reduce its price
- Cooperation is difficult to establish if players interact only once (one-shot game)
- Only Nash-equilibrium is (low, low).

Why do we observe cartels (cooperation) in real life?

- Players in real life do not interact only once, they interact more often
- Benefits of cooperation are higher if agents can interact more often

Dynamics in a cartel

If one player deviates, the other player can punish that firm later.

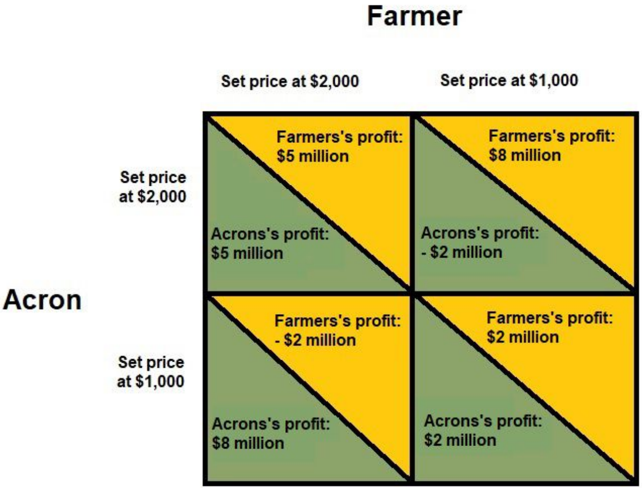
"Tit-for-tat", one successful strategy to increase cooperation:

1. Start cooperative (high, high)
2. Each player does what the other did in the previous round. This solves the cooperation problem,
3. Gain from cheating is 15 (=20-5), the future loss is 2 (=5-3) in each consecutive period.
4. In 8 periods, the gains from cheating are lost.
5. Time horizon matters!

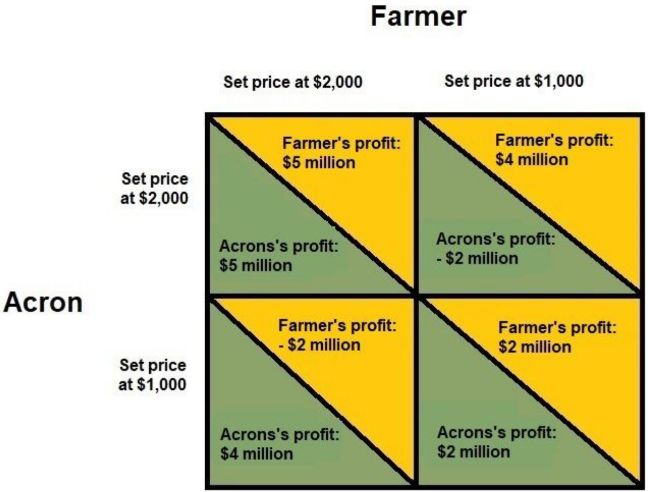
Most-favored-customer clauses

- Version 1
  - If the firm reduces its price subsequent to a purchase, the early customer will get a rebate so that he or she will pay no more than those buying after the price reduction
- Version 2
  - You get a rebate, if you see the product cheaper somewhere else. $\Rightarrow$ **Bestpreisgarantie**
- Looks like a very generous (consumer-friendly) device.
- It can also be a seen as a clever agreement to keep cartel discipline alive.

Payoff Matrix before Most-favored-customer clause



Payoff Matrix after Most-favored-customer clause



Incomplete Information Games (IIG)

Often, players do not have the same information.

- Asymmetric information is summarized as *player types*.
- For example, a type could be "low production costs" or "high production costs"; "cooperative" or "uncooperative", ...
- Players from expectations about the other player's type and consider the associated payoffs.
- Example on next slide.

Different types

A. Barkley managers are tough.

		Allied's strategies	
		Enter the market	Do not enter the market
Barkley's strategies	Fight (price low)	6, 2	8, 3
	No fight (price high)	5, 4	2, 3

B. Barkley managers are soft.

		Allied's strategies	
		Enter the market	Do not enter the market
Barkley's strategies	Fight (price low)	2, 2	3, 3
	No fight (price high)	7, 4	4, 3

Notes: B is an incumbent in a market. A considers to enter the market. A considers two possible scenarios, (Panel A) firm B fights the entrant, (Panel B) firm B tolerates the entrant. If B is tough, A will not enter the market. If B is soft, A will enter. (NB if A does not enter, B will not have to price low!) Figure 12.14 in Allen et al., *Managerial Economics* (8th ed.), p487.

Reputation

Even if firm B is actually soft, it is beneficial to pretend being a tough firm!

- If A believes that B is tough, A will not enter.
- If A does not enter, B has the market.
- Reputation is valuable and firms (managers) invest in reputation.
- However, for reputation to pay off, players need time (repeated vs. single game).
- Reputation is based on a player's history and involves inferring future behavior based on past behavior.

Coordination

In many situations, there are several equilibria.

- Players need coordination to select among the possible equilibria.
- Coordination problems arise from players' inability to communicate, from different strategic models or from asymmetric information.
- Examples:
  - Matching Games
  - "Battle of the Sexes"
  - Stag Hunt
  - First-mover advantage
  - Hawks and Doves

Product Coordination

		Allied's strategies	
		Produce for consumer market	Produce for industrial market
Barkley's strategies	Produce for consumer market	0, 0	7, 7
	Produce for industrial market	12, 12	0, 0

Notes: There are two Nash equilibria. Both firms prefer (12,12) to (7,7). If firms are in (7,7), although they prefer (12,12), they will not deviate unilaterally. The firms need to coordinate their behavior. Figure 12.15 in Allen et al., *Managerial Economics* (8th ed.), p490.

Battle of the Sexes

		Allied's strategies	
		High-end product	Low-end product
Barkley's strategies	High-end product	0, 0	11, 6
	Low-end product	6, 11	0, 0

Notes: The name of the game is from an old framing of a man and a women who have different preferences (football vs. opera), however, both prefer to spend time together. In this situation, there are two Nash-equilibria, however, it is not clear which one will be reached. Figure 12.16 in Allen et al., *Managerial Economics* (8th ed.), p491.

Stag Hunt

	A	
B	Old standard	New standard
Old standard	(6,6)	(6,0)
New standard	(0,6)	(12,12)

Notes: This game goes back to French philosopher Rousseau, two poachers either go for the hare (small prize) or go for the stag (large prize). Catching the stag is only possible if both poachers coordinate their hunting of the stag. Going for the large prize requires coordination (or trust in the other player). Figure 12.17 in Allen et al., *Managerial Economics* (8th ed.), p491.

First-mover's advantage

		Allied's strategies	
		Produce superior product	Produce inferior product
Barkley's strategies	Produce superior product	25, 50	110, 70
	Produce inferior product	30, 140	20, 30

Notes: A and B invest in new technology to produce a new, superior product. Once one firm has the new technology, the other firm can only use the old technology (patent). A's marginal benefit from having this technology first is (30,140) vs (110,70), i.e., 70. B's marginal benefit is 80. This suggests that B moves first. Figure 12.18 in Allen et al., *Managerial Economics* (8th ed.), p492.

Hawks and Doves

	A	
B	Hawk	Dove
Hawk	(-1,-1)	(10,0)
Dove	(0,10)	(5,5)

Notes: Game theory has been used to analyze e.g., political scenarios. If players are prone to fight ("hawks"), conflict is inevitable. If players avoid fights ("doves"), loss from conflict is avoided. Coordination is required for selection of the "good" equilibrium. Figure 12.19 in Allen et al., *Managerial Economics* (8th ed.), p493.

Zero-sum Games

	A		
B	a	b	c
a	(-5,5)	(20,-20)	(-22,22)
b	(-3,3)	(7,-7)	(4,-4)
c	(-4,4)	(-6,6)	(17,-17)

Notes: The gain of one player is equal to the loss of the other player. These situations are known as "zero-sum games". Sometimes, mature markets are seen as zero-sum situations, increasing the market share of one firm reduces the market share of the other firms. Figure 12.20 in Allen et al., *Managerial Economics* (8th ed.), p494.