Internationalization strategies of multi-product firms: The role of technology

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Abstract

High-performance firms typically have two features in common: i) they produce in more than one country and ii) they produce more than one product. In this paper, we analyze the internationalization strategies of multi-product firms at the product-level. We find that the most productive firms sell core varieties via foreign direct investment (FDI) and export products with intermediate productivity. Shocks to trade costs and technology affect the endogenous decision to export or produce abroad at the product-level and, in turn, the relative productivity between parents and affiliates.

JEL codes: F12, F23, L25, L11

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1 Introduction

In international economics, one striking pattern emerges: internationalisation is for the few.\textsuperscript{1} Many empirical studies show that international activity is concentrated in a small share of very large firms. These companies successfully compete in international markets because they are the most productive firms, spend most on R&D activities, and have the highest skilled workforce for whom they pay the highest wages. Among other characteristics, these high-performance firms typically have two features in common. First, they are multinationals, running affiliates in many countries around the world.\textsuperscript{2} Second, these firms produce multiple products and contribute to a large extent to the product variety in the world economy.\textsuperscript{3} The similarities between the documented stylized facts on multinationals on the one hand, and multi-product firms on the other hand, are striking, yet only few studies have analyzed multinational multi-product firms in a unified framework so far.\textsuperscript{4}

In this paper, we analyze the internationalization strategies of multi-product firms at the firm-product level. In doing so, we focus on the role of a firm’s production technology for the optimal mode of serving consumers. Firms are characterized by a flexible manufacturing technology and may decide on the optimal mode of internationalization for each of their products. As firms produce multiple varieties with heterogeneous productivities, differential strategies will be optimal for the various products. In particular, we ask the following questions: Which goods are productive enough to be sold on foreign markets? Where are those goods produced: abroad via horizontal foreign direct investment (FDI) or at home, to be exported to the foreign market? What is the role of globalization and technology shocks in these decisions? And, finally, how do such shocks affect the relative plant-level productivities of parents versus affiliates in multinational firms?

Following the standard literature, firms choose to produce a given product abroad, if their gain from avoiding trade costs offsets their greater fixed cost of production (proximity-concentration trade-off).\textsuperscript{5} The relative size of the gain depends importantly on the market share of the firm’s product. In analogy to Helpman et al. (2004), the most productive firms choose multinational

\textsuperscript{1}See, for instance, the respective chapter on European firms in Mayer and Ottaviano (2008).
\textsuperscript{2}See, e.g., the recent surveys on multinationals by Yeaple (2013b) and Antrás and Yeaple (2014).
\textsuperscript{3}For empirical evidence on the dominance of multi-product firms, see, e.g., Bernard et al. (2007), Bernard et al. (2009), Bernard et al. (2010, 2011), Broda and Weinstein (2010), Goldberg et al. (2010).
\textsuperscript{4}Important exemptions from this are Baldwin and Ottaviano (2001), Yeaple (2013a), and Tintelnot (2017), which are discussed in greater detail below.
\textsuperscript{5}See, for example, Horstmann and Markussen (1992), Brainard (1993, 1997), Markussen and Venables (2000), Markussen and Maskus (2002), Helpman et al. (2004).
production to serve foreign consumers, however, they do not do so for their entire product range. In contrast to most existing models on the proximity-concentration trade-off, we allow for a second source of heterogeneity that affects a product’s market share. Besides between-firm heterogeneity à la Melitz (2003), we introduce within-firm heterogeneity between products. Following Eckel and Neary (2010), firms operate with a flexible manufacturing technology such that the marginal cost of a product is increasing in its distance from the firm’s core competence. Firms may endogenously decide on the range of products being produced, and the rank of a product within the portfolio of a firm will determine the optimal way of serving consumers abroad. We find that core products are sold via FDI, while products of an intermediate productivity are exported. As a direct consequence of that, foreign affiliates show a higher level of productivity at the plant-level compared to their parent firms. This result differs importantly from a model with single-product firms, as in Helpman et al. (2004), where affiliate and parent firms have the same productivity. The reason behind this difference is that, in our case, the foreign plant only produces a subset of the products that are produced in the parent plant. Since FDI is only profitable for core varieties, plant-level productivity is higher in the foreign affiliate.

As another difference compared to the model in Helpman et al. (2004), in our model the most productive firms rely on both strategies, that is they both export and invest abroad. This result is in line with evidence from Spanish firm-level data, where we find that the share of firms with international engagement via both FDI and exporting is increasing in firm productivity.

Having established the endogenous choice of the different modes of market entry at the product-level, we further investigate the role of technology in the internationalization decision. In particular, we analyze the impact of production flexibility on the relative sales in different modes at the firm level. For example, it could be that some firms operate a more flexible technology, where the introduction of new varieties is associated with a lower increase in marginal costs. The flexibility of technology might also vary across industries. We find that more flexible firms have greater domestic sales as well as greater sales in both exports and FDI. Moreover, an increase in production flexibility increases the share of export sales for multinationals, whereas the opposite is true for firms that export but do not engage in FDI. In these firms, greater flexibility decreases the share of export

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7 These findings fit remarkably well with empirical evidence on U.S. multinationals first pointed out by Yeaple (2013a).
sales. The reason behind this result is that, as flexibility increases, firms skew their sales away from their best-performing products, that is products sold via FDI (exports) in case of firms with high (medium) productivity.

As a direct implication from our analysis, we find that any shock (such as globalization, or technology) that affects the endogenous FDI/export decision changes the productivities of both affiliate and parent firms. Moreover, these shocks also determine whether profits of the most profitable core varieties are recorded at home (in case of exports) or abroad (in case of FDI). This is crucial from a policy perspective, as it defines the location where corporate taxes have to be paid. In addition, it determines the extent to which home workers or foreign workers are involved in production.

Our paper is related to two broad strands of the recent literature in international economics. First, it contributes to the literature on multi-product firms, which has been rapidly increasing in the past few years due to the availability of detailed product-level data. Based on novel stylized facts from empirical work, a growing number of theoretical contributions implements the analysis of multi-product firms in existing models of international trade (see, for example, Feenstra and Ma (2008), Bernard et al. (2010, 2011), Eckel and Neary (2010), Dhingra (2013), Qiu and Zhou (2013), Yeaple (2013a), Mayer et al. (2014), Nocke and Yeaple (2014), Flach and Irlacher (2018), and Arkolakis et al. (2020)). They typically investigate the product scope within multi-product firms (intra-firm extensive margin) as an important margin of adjustment to changes in market conditions. In contrast to our paper, their focus is mainly on the effect of trade liberalization on export scope, whereas the role of FDI is not included in the analysis.

Second, our paper contributes to the literature that analyses firm’s optimal mode of foreign market access, distinguishing between multinational production and exporting as two different choices based on the so-called proximity-concentration trade-off (see, for example, Horstmann and Markusen (1992), Brainard (1993, 1997), Markusen and Venables (2000), Markusen and Maskus (2002), Helpman et al. (2004)). In particular, similar to Helpman et al. (2004), we focus on the role of firm heterogeneity for individual market access strategies and the resulting pattern of aggregate international production and trade. However, we extend Helpman et al. (2004), who focus on single-product

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8 A number of empirical contributions document the dominance of large multi-product firms in international markets (see, for example, Bernard et al. (2007), Bernard et al. (2009), Bernard et al. (2010, 2011), Broda and Weinstein (2010), and Goldberg et al. (2010)). Moreover, Bernard et al. (2010) and Broda and Weinstein (2010) show that most product creation and destruction happens within existing firms, which has important potential implications for aggregate product scope and welfare.

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firms, in allowing for firms to produce more than one product. In this framework, we can analyze
optimal product scope together with optimal market access at the firm-product level. Importantly,
we distinguish between two different sources of heterogeneity: between-firm heterogeneity in (core)
productivity and within-firm heterogeneity across products.

Our paper is most closely related to papers that combine the two strands of the literature
discussed above. In an early contribution, Baldwin and Ottaviano (2001) build a model in an oli-
gopolistic setting where multi-product firms reduce inter-variety competition (i.e. cannibalization)
by relocating some varieties abroad. The driving force in their model is similar to the reciprocal
dumping model in Brander and Krugman (1983) and fundamentally different to the logic in our
analysis. Yeaple (2013a) provides an interesting set of novel stylized facts on multinational multi-
product firms consistent with our predictions. However, in contrast to our model, his focus is
not on production flexibility, but on managerial expertise as a scarce resource that has to be sub-
divided across products in different locations. Firms differ both in their endowments of managerial
expertise and in their efficiency of transferring this expertise to foreign affiliates. The analysis in-
vestigates how these two sources of managerial heterogeneity affect the product range as well as
the exports/FDI mix of multi-product firms. Tintelnot (2017) investigates the determinants of the
location and production of multinational firms when foreign affiliates of multinationals may serve
as export platforms. Using French firm-product level data, Bricongne et al. (2019) analyze whether
FDI and exports are complements or substitutes. They find that firms that do FDI export more,
confirming the predominant result in the literature. However, consistent with our model, they also
find that this is not true for core products, in particular for the most productive firms in countries
with strong demand.

The structure of the remainder of this paper is as follows. In Section 2, we provide an empirical
motivation for our subsequent analysis. In Section 3, we describe our theoretical model and derive
our main results. Section 4 concludes.
2 Empirical motivation

To motivate our theoretical framework, we use the Spanish Encuesta sobre Estrategias Empresariales (ESEE), a representative sample of Spanish manufacturing firms with more than 10 employees.\textsuperscript{9} The data set is extremely rich and contains information on both export and FDI activities of firms. In the following, we are interested in the relationship between firm productivity and the modes of serving foreign markets. We find that the share of firms with an international engagement via exports and/or FDI increases in firm productivity (see Figure 1).

![Figure 1: Share of firms doing exports and/or FDI across productivity deciles](image)

In Figure 1, (labor) productivity is measured as value added per hour worked by employees. Based on this measure, firms are grouped into deciles, normalized by industry and year. That is, we explore variation across firms within the same industry/year combination. The graph shows that the composition of firms by mode of access changes along the productivity distribution. The share of firms engaged in exporting increases steadily up to the 6th productivity decile, then remains at

\textsuperscript{9}It is a panel data set, which runs since 1990 and has a high response rate among repeatedly interviewed firms. Between 1,500 and 2,000 firms are interviewed each year. For the purpose of our data exploration, we make use of the waves of the years 2002, 2006, 2010, and 2014. Among others, these data have been used by Guadalupe et al. (2012), Garicano and Steinwender (2016), as well as Koch and Smolka (2019).
roughly at the same level up to the 8th decile, and finally declines for the most productive firms. In contrast, hardly any firms are engaged in FDI in the bottom half of the productivity distribution, but the share rises quickly thereafter. Importantly, the entire increase is driven by firms doing both exports and FDI, while the share of firms with only FDI (and no exports) is extremely low along the entire distribution.

**Stylized facts** Relating firm productivity to the differential modes of serving foreign markets, we find the following facts:

- the share of producers that only export is highest for medium productive firms,
- the share of firms that rely on both exports and FDI strictly increases in productivity,
- the share of firms that only rely on FDI is close to zero across all deciles.

While the first fact above is consistent with the standard model of Helpman et al. (2004), the remaining two are not. A number of studies analyze the relationship between exports and FDI at the firm level. For US firms, Lipsey and Weiss (1984) and Desai et al. (2005) show that increased production in foreign affiliates is related to larger parent exports. Similarly, Lipsey et al. (2000) and Head and Ries (2001) find a positive correlation between exports and foreign production for Japanese firms. These studies indicate that firms may rely on both strategies in serving a particular region or country.\(^{10}\) There are several possible reasons. First, foreign production may promote the sales of intermediate goods in firms that are vertically integrated.\(^{11}\) Second, firms may serve a given foreign market via FDI at one point in time and via exports at another (see, e.g., the dynamic models of Conconi et al. (2016) and Gumpert et al. (2020)). Third, firms may produce some products abroad and export others (Yeaple (2013a)). The latter fits well with evidence at the product level, where exports and FDI have been shown to constitute substitutes rather than complements (see, e.g., Blonigen (2001), Swenson (2004), Bricongne et al. (2019)). In the following, we pursue this idea and develop a model to determine optimal modes of market access on the firm-product level. We find that the most productive firms sell their most productive products via FDI.

\(^{10}\)Lipsey et al. (2000) control for the region of destination. Gumpert et al. (2020), referred to below, document the coexistence of FDI and exports in Norwegian firms at the firm-country-of-destination-year level.

\(^{11}\)Head and Ries (2001) find some evidence for this.
and the products with intermediate productivity via exports. Firms with medium productivity sell their most productive products via exports. They do not engage in FDI. Firms with low productivity sell only at home. Our findings are consistent with the stylized facts above and provide a novel explanation for the relative importance of exports and FDI, as well as the relative performance of parent firms and affiliates.\footnote{Coşar et al. (2018) study the automobile industry using a worldwide dataset that contains the assembly plant locations of 598 car models. They find that 43\% of the models are assembled in more than one country and account for 64\% of total revenue. This indicates that, in line with our theoretical predictions, foreign assembly takes place in particular for core varieties within the firm.}

3 The model

We extend the model of Helpman et al. (2004) to explain how heterogeneous multi-product firms choose to enter foreign markets, and to explore the role of production technology in these decisions. As in the standard model, there is heterogeneity in the productivity between firms. In addition, there is heterogeneity in the productivity between products within firms due to flexible manufacturing à la Eckel and Neary (2010). As a result, the model features two sources of firm heterogeneity: first, in absolute core productivity (between-firm heterogeneity) and, second, within-firm heterogeneity between products due to flexible manufacturing. Firms decide whether to enter the market or not, how many goods to produce, where to supply these goods, and whether to serve a foreign market via exports or FDI.\footnote{We abstract from the possibility of exports by foreign affiliates (see also Helpman et al. (2004)).} Importantly, the last two decisions are made at the product level. We find that there is firm dispersion in total sales, in product scope (the number of products sold domestically and abroad via exports or FDI), and in the decision of whether to supply a given product to a foreign market via exports or FDI (or not at all). In particular, in line with our empirical motivation, we find that the most productive firms choose to serve foreign markets through both FDI and exports.

3.1 Consumers

We consider a world of two symmetric countries $i$ and $j$. Both countries use labor to produce goods in $M + 1$ sectors. We take the homogeneous good as the numeraire and assume that both countries always produce it with one unit of labor per unit output. As a result, the wage rate is equal to one in both countries. The remaining $M$ sectors are characterized by monopolistic competition.
and produce differentiated varieties with a constant elasticity of substitution $\sigma > 1$. Consumers in country $i$ spend a share $\beta_m$ of their income $E_i$ on goods from sector $m$ and the remaining fraction $1 - \sum_{m=1}^{M} \beta_m$ on the outside good. Each country’s representative consumer has preferences described by the following utility function:

$$U = \left(1 - \sum_{m=1}^{M} \beta_m\right) \log z + \sum_{m}^{M} \beta_m \log C_m,$$  

(1)

where $z$ is the consumed quantity of the homogeneous good, and $c_{ijgm}(\omega)$ is the consumed quantity of variety $\omega$ of product $g$ from sector $m$.\(^{14}\) Here, $\omega$ indexes varieties of product $g$ supplied from country $j$ to country $i$ and $\Omega_{ijgm}$ is the endogenous set of these varieties.\(^{15}\) Consumers maximize utility subject to the budget constraint $z + \sum_{m=1}^{M} \int_{g=0}^{\infty} \int_{j=1}^{2} \int_{\omega \in \Omega_{ijgm}} p_{ijgm}(\omega)c_{ijgm}(\omega)d\omega dg \leq E_i$. In the following, we focus on a sector-by-sector analysis and drop the subscript $m$, as well as the subscripts $g$ and $j$, unless required. Utility maximization implies that product demand in country $i$ in any particular sector is given by:

$$c_i(\omega) = \beta E_i P_i^{1-\sigma} p_i(\omega)^{-\sigma} \equiv B_i p_i(\omega)^{-\sigma},$$

(3)

where

$$P_i = \left(\sum_{m=1}^{M} \int_{g=0}^{\infty} \int_{j=1}^{2} \int_{\omega \in \Omega_{ijgm}} p_{ijgm}(\omega)^{1-\sigma} d\omega dg\right)^{\frac{1}{1-\sigma}}.$$  

3.2 Firms

Starting a firm in a differentiated sector requires a fixed cost of entry $f_e$. Firms are heterogeneous in productivity and draw a firm specific efficiency parameter $\phi \in [0, \infty]$ from a cumulative distribution function $F(\phi)$ that is the same across countries. After a firm has paid the fixed entry cost, it observes its core productivity, $\phi$, and decides whether to exit or remain in the market. In case it

\(^{14}\)Sectors are defined such that firms produce all their products within the same sector.

\(^{15}\)It is assumed here for simplicity that the elasticity of substitution across varieties within products is the same for all products and equal to the elasticity of substitution across products. Moreover, each firm produces at most one variety of product $g$ (see also, e.g., Bernard et al. (2011)).
remains, it also decides how many products to sell in a given country and – if it decides to sell a given product also in a foreign country – whether to do so via exports or via FDI. Serving the domestic market requires a fixed cost \( f_d \) per variety. Serving a foreign market via exports requires a fixed cost \( f_x \) and, in addition, for each product that is exported, firms face common (across firms and products) iceberg trade costs, so that \( \tau_{ij} > 1 \) units must be shipped from country \( i \) to country \( j \) for one unit to arrive.\(^{16}\) Firms that serve a foreign market via FDI avoid variable trade costs but have to pay higher fixed cost \( f_m \). Importantly, in contrast to the fixed market entry cost \( f_e \), the fixed costs \( f_d, f_x, \) and \( f_m \) are product-specific. We follow Helpman et al. (2004) and assume the following parameter restriction:

**Assumption 1** \( f_d < \tau^{\sigma-1} f_x < f_m \).

Throughout our analysis, we assume that both fixed as well as variable trade costs are identical for all firms in a given sector but may vary across sectors. This allows us to compare results across sectors with a different cost structure.

### 3.2.1 Technology

Following Eckel and Neary (2010), we assume that firms operate with a flexible manufacturing technology, such that introducing additional varieties is associated with a lower productivity. Firms produce each product according to a linear production technology using labor with product-specific efficiency \( \phi_g \). Marginal costs are constant for a given product, but increase in distance from a firm’s core competence:

\[
\phi_g \equiv \frac{\phi}{h(g)} \quad \text{with} \quad h'(g) > 0, \quad h(1) = 1. \tag{4}
\]

To derive closed form solutions, we follow Arkolakis et al. (2020) and parameterize the cost function as follows:

\[
h(g) = g^\alpha, \quad \alpha \in [0, +\infty). \tag{5}
\]

The parameter \( \alpha \) plays an important role, as it governs the flexibility of the production process. Smaller values of \( \alpha \) imply a higher flexibility of production as marginal costs increase only moderately with distance from a firm’s core competence. In principle, this parameter could vary between firms

\(^{16}\)There are no transport costs for products that are sold domestically \((\tau_{ii} = 1)\).
such that firms differ in the flexibility of production. In the main part of this paper, we do not need to take a stand on whether $\alpha$ varies between firms within an industry or between industries. In the Appendix, we solve for the general equilibrium and assume that, similarly to the fixed costs of production, the flexibility of production technology is sector-specific (i.e., identical for all firms within a given sector). There, comparative statics with respect to $\alpha$ should be interpreted as comparing results across sectors with different production flexibility.\footnote{Indeed, we find a substantial variation across industries in the use of flexible manufacturing systems in the ESEE data. Specifically, we make use of the following survey question: “State whether the production process uses any of the following systems: 4. Combination of some of the above systems (i.e. 1. Computer-digital machine tools; 2. Robotics; 3. Computer-assisted design) through a central computer (CAM, flexible manufacturing systems, etc.)”. The share of firms making use of these systems varies between less than 10% for the industry “Leather, fur and footwear” and almost 50% for the industry “Vehicles and accessories”.}

In analogy to Arkolakis et al. (2020), we define an efficiency index at the plant-level. The average product efficiency of a plant producing a total of $G$ products (not taking into account fixed costs of production) is given by:

$$H(G) = \left( \frac{1}{G} \sum_{g=1}^{G} g^{-\alpha(\sigma-1)} \right)^{-\frac{1}{\sigma-1}}.$$ \hspace{1cm} (6)

This index decreases in the number of varieties $G$, and the drop in average efficiency for each additional variety is larger the greater the value of $\alpha$ (see Figure 2). Note that this index converges to one for large values of the technology parameter $\alpha$ or the elasticity of substitution $\sigma$, which reflects the scenario where almost all sales are concentrated in the core variety.

### 3.2.2 Optimal firm behavior

Each firm chooses product prices to maximize profits under monopolistic competition given consumer demand (3) and productivity $\phi/h(g)$.\footnote{Monopolistic competition implies that the price of each product variety can be chosen independently of the prices of other varieties. That is, there is no strategic interaction, unlike in, e.g., Eckel and Neary (2010).} This results in identical markups $\sigma/(\sigma - 1)$ over marginal costs:\footnote{Optimal prices are derived from the expressions for total firm operating profits by mode of production given in Appendix A.1.}

$$p_{ii} = \frac{\sigma}{\sigma - 1} \frac{g^\alpha}{\phi} \quad \text{and} \quad p_{ij} = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} g^\alpha}{\phi}.$$ \hspace{1cm} (7)

In the next step, we derive per-variety profits for domestic sales, exports as well as FDI sales. Before we do that, it is convenient to define the operating profit of the core product of a firm with
Figure 2: Production flexibility

productivity $\phi$:

$$\tilde{\pi}(\phi) = A\phi^{\sigma-1},$$

(8)

where $A \equiv \frac{1}{\sigma} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} B$ denotes the mark-up adjusted revenue shifter (identical across the two countries due to the symmetry assumption). Obviously, firms with a higher core productivity $\phi$ are more profitable in their core variety $g = 1$. Substituting optimal prices into the firm’s profit functions, we obtain the respective profits for the different modes of serving a market (domestically or abroad via exports or FDI):

$$\pi_d(g) = \tilde{\pi}(\phi)g^{-\alpha(\sigma-1)} - f_d,$$

(9)

$$\pi_x(g) = \tilde{\pi}(\phi)\tau^{1-\sigma}g^{-\alpha(\sigma-1)} - f_x,$$

(10)

$$\pi_m(g) = \tilde{\pi}(\phi)g^{-\alpha(\sigma-1)} - f_m.$$  

(11)

Equations (9)-(11) indicate that in any mode of market entry, per-variety profits decrease in distance from the core competence. This drop in profitability is more pronounced the lower is the flexibility of production (higher values of $\alpha$).
Productivity cutoffs The profit equations above determine the survival cutoff ($\phi^*_d$) as well as the minimum productivities for selling the core product abroad via exports ($\phi^*_x$) or FDI ($\phi^*_m$). The first two cutoffs are the solutions to $\pi_d(1) = 0$ and $\pi_x(1) = 0$, respectively. The cutoff for FDI is the solution to $\pi_x(1) = \pi_m(1)$. The solutions for the three cutoffs are given by:

$$
\phi^*_d = \left( \frac{f_d}{A} \right)^{\frac{1}{\sigma-1}}, \tag{12}
$$

$$
\phi^*_x = \left( \frac{f_x}{A(1-\tau^{1-\sigma})} \right)^{\frac{1}{\sigma-1}}, \tag{13}
$$

$$
\phi^*_m = \left( \frac{f_m - f_x}{A(1-\tau^{1-\sigma})} \right)^{\frac{1}{\sigma-1}}. \tag{14}
$$

Our parameter restrictions according to Assumption 1 ensure that $\phi^*_d < \phi^*_x < \phi^*_m$.\footnote{The first (second) inequality ensures that there are firms that sell in the domestic market (abroad via exports) only (compare Helpman et al. (2004)).} Hence, low-productivity firms sell their core product only in the domestic market, medium-productivity firms sell it abroad via exports, and high-productivity firms sell it abroad via FDI.

In general equilibrium, the cutoffs $\phi^*_d$, $\phi^*_x$ and $\phi^*_m$, together with the demand level $A$, are solutions to equations (12)-(14) in combination with the free entry condition (see Appendix A.2). In the following, we focus on firm-level adjustments in an industry equilibrium with a given number of firms. We postpone the general equilibrium analysis to the Appendix of this paper.\footnote{Note that, given the symmetry assumptions we made, the two countries share the same cutoffs and demand levels in general equilibrium. As long as wages are equalized, this results also holds for different country sizes. As discussed in Helpman et al. (2004), the larger country attracts a larger measure of entrants.}

Optimal scope in each mode We define the scope of products sold domestically and abroad via exports or FDI as $G_d$, $G_x$, and $G_m$. Given our parameter restrictions on fixed costs, the minimum operating profit required to cover fixed costs is smallest for products that are sold domestically, and smaller for products that are exported compared to products that are sold via FDI. As a product’s price increases and, therefore, its revenue\footnote{This is because demand is elastic ($\sigma > 1$).} and operating profit\footnote{Operating profit is proportional to revenue due to CES preferences.} decrease in distance from the core competence, products that are closest to a firm’s core will be sold via FDI (given that the firm undertakes any FDI at all, i.e., conditional on $\phi \geq \phi^*_m$). Products that are further away will be
exported, and the products with the greatest distance from the core will only be sold domestically.\footnote{Products are sold abroad via FDI or exports in addition to being sold domestically.}

Next, we define the marginal products \( g_d \) and \( g_x \in \{0, 1, \ldots\} \), that is, the largest \( g \in \{0, 1, \ldots\} \) such that \( \pi_d(g) \) and \( \pi_x(g) \) are equal to (or greater than) zero, respectively. Furthermore, the marginal product \( g_m \in \{0, 1, \ldots\} \) is the largest \( g \in \{0, 1, \ldots\} \) such that \( \pi_m(g) \geq \pi_x(g) \). The determination of marginal products is graphically illustrated for the example of two firms with different productivity levels in Figure 3. They occur at the point where operating profits of the core product are equal to the respective combined incremental scope costs:

\[
\pi_d(g) = 0 \Rightarrow \overline{\pi}(\phi) = f_d g^{\alpha(\sigma-1)} \quad \text{and} \quad \pi_x(g) = 0 \Rightarrow \overline{\pi}(\phi) = f_x g^{\alpha(\sigma-1)} \tau^{\sigma-1}.
\]

The upward sloping loci for domestic and export scope in Figure 3 represent the incremental scope costs. Here, the comparatively larger slope for the export locus follows from Assumption 1. In our example, the firm with intermediate productivity (\( \phi_1 > \phi^*_x \)) exports its core varieties (1 and 2) and additionally produces three more varieties for domestic sales. The FDI locus is determined by the following equation:

\[
\pi_x(g) = \pi_m(g) \Rightarrow \overline{\pi}(\varphi) = \frac{(f_m - f_x) g^{\alpha(\sigma-1)}}{(1 - \tau^{1-\sigma})}.
\]

It has the comparatively largest slope, by Assumption 1. Operative profits of the core variety of the firm with productivity \( \phi_2 > \phi^*_m \) are above the FDI locus such that this firm prefers multinational production for its most productive varieties (1 and 2) and exports its products with intermediate productivity. Here, the firm exports three varieties and has an overall product range of ten varieties. We summarize this analysis in our first proposition, which is in line with our empirical motivation.

**Proposition 1** Firms with productivity \( \phi > \phi^*_m \) engage in both multinational production and exporting. They sell core products via FDI and export products with an intermediate productivity.

Solving for the marginal products in each mode, we derive:

\[
g_d = \int \left\{ \frac{A \phi^{\sigma-1}}{f_d} \right\}^{\frac{1}{\alpha(\sigma-1)}}, \tag{15}
\]

\[
g_x = \int \left\{ \frac{A \phi^{\sigma-1} \tau^{1-\sigma}}{f_x} \right\}^{\frac{1}{\alpha(\sigma-1)}}, \tag{16}
\]
Figure 3: Exports versus FDI at the product-level

\[
g_m = \text{int}\left\{ \left( \frac{A\phi^{\sigma-1}(1-\tau^{1-\sigma})}{f_m-f_x} \right)^{\frac{1}{\alpha(\sigma-1)}} \right\}.
\]

(17)

The total range of products is given by \( G_d = g_d \), \( G_x = g_x - g_m \), \( G_m = g_m \), respectively (see Figure 3). Using (12)-(14), we express marginal products in terms of the cutoff productivity level for the core product:

\[
g_d = \left( \frac{\phi}{\phi_d^*} \right)^{\frac{1}{\alpha}}, \quad g_x = \left( \frac{\phi}{\phi_x^*} \right)^{\frac{1}{\alpha}}, \quad \text{and} \quad g_m = \left( \frac{\phi}{\phi_m^*} \right)^{\frac{1}{\alpha}}.
\]

(18)

Note that, for any strictly positive product scopes, we have \( g_m < g_x < g_d \), since \( \phi_d^* < \phi_x^* < \phi_m^* \) by Assumption 1. It follows that \( G_d > G_x + G_m \), such that varieties sold abroad are a subset of all varieties within the portfolio of a firm. Marginal varieties are only sold domestically, since they are not profitable enough to be sold abroad.

In a next step, we investigate the effect of production flexibility on the optimal product range in each mode. In partial equilibrium (i.e. conditional on given cutoffs), a more flexible production (lower levels of \( \alpha \)) increases optimal scope in each mode since the marginal product is getting more efficient in production. In Figure 3, this corresponds to an outward rotation of the respective loci for all three modes. We summarize these results in the next proposition.
Proposition 2 In any given mode of entry, more productive firms produce a greater range of products. For given cutoff productivities, product scope in all modes increase in production flexibility (smaller values of \( \alpha \)).\(^{25}\)

To conclude this section, we briefly investigate the effects of trade liberalization on the product scopes of multi-product firms. In Appendix A.2, we show that, in general equilibrium, lower variable trade costs (\( \tau \)) or lower fixed costs of exporting (\( f_x \)) increase domestic competition and, therefore, the survival cutoff \( \phi_d^* \). According to equation (18), any shock that increases the survival productivity cutoff (\( \phi_d^* \)) induces firms to focus more on their core varieties and reduce total product scope.\(^{26}\)

3.3 Exports versus FDI at the firm-product level

In this section, we derive results with respect to the share of FDI and export products and sales at the firm level. This disaggregate analysis of optimal market entry strategies allows us to compare our results for different types of firms (i.e. multinationals versus exporters only). In addition, it allows us to compare plant-level productivities of multinational affiliates to their respective parent firms.

Share of FDI products In the following, we determine the share of a firm’s FDI products in the total number of its varieties sold domestically and abroad. Using the expressions for marginal products (18) and substituting for the cutoff productivities (12)-(14), we derive:

\[
\frac{G_m}{G_d} = \frac{g_m}{g_d} = \left( \frac{\phi_d^*}{\phi_m^*} \right)^{\frac{1}{\alpha}} = \left( \frac{1 - \tau^{1-\sigma} f_d}{f_m - f_x} \right)^{\frac{1}{\alpha(\sigma-1)}}, \tag{19}
\]

\[
\frac{G_m}{G_x + G_m} = \frac{g_m}{g_x} = \left( \frac{\phi_x^*}{\phi_m^*} \right)^{\frac{1}{\alpha}} = \left( \frac{f_x}{f_m - f_x} \frac{(1 - \tau^{1-\sigma})}{\tau^{1-\sigma}} \right)^{\frac{1}{\alpha(\sigma-1)}}. \tag{20}
\]

Equations (19)-(20) show that the share of FDI products does not depend on firm productivity (\( \phi \)): conditional on being a multinational firm, the share of FDI products is constant across firms with different productivities. The FDI share does, however, depend on the flexibility of the production

\(^{25}\)This direct negative effect of \( \alpha \) on product scope is counteracted by an indirect positive effect, since product scope decreases, as \( \alpha \) becomes smaller, via an increase in the cutoff productivities in general equilibrium (i.e., \( \partial \phi_d^*/\partial \alpha < 0 \), see Appendix A.2).

\(^{26}\)This confirms a well-known result in the literature on multi-product firms saying that, as trade costs fall, competition in the domestic market rises such that firms drop products with the highest marginal costs (see, for instance, Eckel and Neary (2010), Bernard et al. (2010) or Mayer et al. (2014)).
technology. Comparing two industries (or firms) that only differ in the flexibility of production (the parameter $\alpha$), equations (19) and (20) suggest that the share of FDI products is higher in the industry (firm) with a lower flexibility of production (higher values of $\alpha$).\footnote{Note that the FDI share (as well as other relative measures of firm performance that follow) does not depend on the economy-wide productivity cutoff $\phi^*_d$. Therefore, we do not need to consider general equilibrium effects here, and can directly compare firms with different values of $\alpha$. In the presence of general equilibrium effects, however, we could only allow $\alpha$ to vary at the industry level.} Intuitively, a lower flexibility of production implies that marginal costs increase faster in distance from the core competence, such that marginal varieties exhibit a greater cost disadvantage compared to core varieties. Since FDI products are closest to the core competence, they represent a higher share in total products. Results are summarized in the following proposition.

**Proposition 3** Consider a firm that engages in FDI. The share of products sold via FDI i) decreases in the flexibility of production (smaller $\alpha$), ii) is constant in firm productivity ($\phi$).

**Proof.** Differentiating Equation (19) with respect to $\alpha$, we derive: 
\[
\frac{\partial \left( \phi^*_d \right) \frac{1}{\alpha}}{\partial \alpha} = -\frac{\ln \left( \frac{\phi^*_d}{\phi^*_m} \right)}{\phi^*_d^2 / \alpha^2} > 0,
\]
which follows from $\ln \left( \frac{\phi^*_d}{\phi^*_m} \right) < 0$ due to $\phi^*_d < \phi^*_m$. Note that this result also holds in general equilibrium. The indirect effect of $\alpha$ on the cutoffs in general equilibrium affects both cutoffs in the same way and, hence, cancels out when considering relative cutoffs. ■

Note that a firm’s FDI share also varies with the costs of exporting and FDI. Equations (19)-(20) show that it increases in $f_x$ and $\tau$ and decreases in $f_m$.

**Share of exported products** Next, we analyse a firm’s share of exported products in its total number of products. We distinguish between two different types of firms: Firm (1) is a multinational enterprise and firm (2) is an exporter only. The respective shares of exported products are given by:

\[
\frac{G_x^{(1)}}{G_d^{(1)}} = \frac{g_x - g_m}{g_d} = \left( \frac{\phi^*_d}{\phi^*_x} \right)^{\frac{1}{\alpha}} - \left( \frac{\phi^*_d}{\phi^*_m} \right)^{\frac{1}{\alpha}},
\]

\[
\frac{G_x^{(2)}}{G_d^{(2)}} = \frac{g_x}{g_d} = \left( \frac{\phi^*_x}{\phi^*_m} \right)^{\frac{1}{\alpha}}.
\]

From (21)-(22), it follows that the qualitative effect of production flexibility on export share differs by firm type. Our model predicts that a lower flexibility of production within an industry is associated with a lower share of exported products in multinationals but a higher share of exported
products within firms that export only (and do not engage in FDI at the same time). This is because a lower flexibility of production results in a drop of the marginal varieties of a firm, increasing the share of the most productive varieties within the firm’s portfolio. Hence, regarding exporting firms, the share of exported varieties is greater when the flexibility of production is lower. For multinationals, this is different because exported varieties are less efficient than the (core) varieties for which the firm chooses multinational production. Thus, while for exporting-only firms the share of exported products is greater when the flexibility of production is lower, the opposite is true for multinational firms. We summarize these insights in the following proposition.

**Proposition 4** The flexibility of production affects the share of exported products in total firm products in different ways for multinational and non-multinational firms.

i) For exporting-only firms (with productivity $\phi^* \leq \phi < \phi^*_m$), the share of exported products decreases in the flexibility of production (smaller $\alpha$): $\partial (G_x/G_d)/\partial \alpha > 0$.

ii) For FDI firms (with productivity $\phi \geq \phi^*_m$), the share of exported products increases in the flexibility of production (smaller $\alpha$): $\partial (G_x/G_d)/\partial \alpha < 0$.

**Proof.** Differentiating equations (21) and (22) with respect to $\alpha$, we derive:

$$\frac{\partial G^{(1)}_d}{\partial \alpha} = -\frac{1}{\alpha^2} \ln \left( \frac{\phi^*_d}{\phi^*_x} \right) \left( \frac{\phi^*_d}{\phi^*_m} \right)^{\frac{1}{\alpha}} - \ln \left( \frac{\phi^*_d}{\phi^*_m} \right) \left( \frac{\phi^*_d}{\phi^*_m} \right)^{\frac{1}{\alpha}} < 0.$$  

$$\frac{\partial G^{(2)}_d}{\partial \alpha} = -\frac{1}{\alpha^2} \ln \left( \frac{\phi^*_d}{\phi^*_x} \right) \left( \frac{\phi^*_d}{\phi^*_m} \right)^{\frac{1}{\alpha}} > 0.$$  

To see this, note that Assumption 1 implies that $1 > \left( \frac{\phi^*_d}{\phi^*_x} \right)^{\frac{1}{\alpha}} > \left( \frac{\phi^*_d}{\phi^*_m} \right)^{\frac{1}{\alpha}}$. Again, this result also holds in general equilibrium. $\blacksquare$

**Relative sales** Next, we determine the composition of firm sales by domestic, export, and FDI sales. This allows us to compare relative sales by mode of market entry similar to Helpman et al. (2004). The key difference is that we can compare relative sales not only between but also
within firms. Similarly to our analysis above, we will again consider the effect of the degree of (in)flexibility of production ($\alpha$) on the composition of firm sales.

Using the definitions above, domestic and FDI sales of any given product $g$ are given by

$$y_d(\phi) = y_m(\phi) = \sigma f_d \left( \frac{\phi}{\phi_d} \right)^{\sigma-1} g^{-\alpha(\sigma-1)},$$

and export sales are given by

$$y_x(\phi) = \sigma \tau^{1-\sigma} f_d \left( \frac{\phi}{\phi_d} \right)^{\sigma-1} g^{-\alpha(\sigma-1)}.$$

Aggregating (at the level of the firm) over the varieties sold in each mode and using the efficiency index defined in equation (6), we can express total sales in the domestic market and abroad via exports and FDI for a firm with productivity $\phi$ as follows:

$$t_d(\phi) = \sigma f_d \left( \frac{\phi}{\phi_d} \right)^{\sigma-1} H_d(\phi)^{-\sigma^{-1}}, \quad H_d(\phi) \equiv \left( \sum_{g=1}^{g_d} g^{-\alpha(\sigma-1)} \right)^{-\frac{1}{\sigma^{-1}}},$$

$$t_x(\phi) = \sigma \tau^{1-\sigma} f_d \left( \frac{\phi}{\phi_d} \right)^{\sigma-1} H_x(\phi)^{-\sigma^{-1}}, \quad H_x(\phi) \equiv \left( \sum_{g=g_m+1}^{g_x} g^{-\alpha(\sigma-1)} \right)^{-\frac{1}{\sigma^{-1}}},$$

$$t_m(\phi) = \sigma f_d \left( \frac{\phi}{\phi_d} \right)^{\sigma-1} H_m(\phi)^{-\sigma^{-1}}, \quad H_m(\phi) \equiv \left( \sum_{g=1}^{g_m} g^{-\alpha(\sigma-1)} \right)^{-\frac{1}{\sigma^{-1}}}. $$

Note that the terms $H_k(\phi)^{-\sigma^{-1}}$ increase in produce scope $g_k$ ($k \in d, x, m$) and, in turn, product scope weakly increases in $\phi$ according to (15)-(17). Hence, total sales in each mode increase in firm

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28 Note that, assuming that firms draw their core productivity from a Pareto distribution with shape parameter $\kappa$, the ratio of aggregate export sales ($s_x$) to aggregate sales via FDI ($s_m$) in an industry is given by

$$\frac{s_x}{s_m} = \tau^{1-\sigma} \left[ \frac{f_m - f_x}{f_x} \frac{1}{\tau^{\sigma-1} - 1} \right]^{\frac{n-\alpha(\sigma-1)}{\sigma-1}} - 1. $$

This is the same as the corresponding expression in the case of single-product firms (see equation (7) in Helpman et al. (2004)). In consequence, all the comparative statics results with respect to cross-sectoral variation in relative export sales derived for single-product firms continue to hold in a framework with multi-product firms. That is, relative export sales decrease in the costs of exporting, $f_x$, and $\tau$, and increase in the fixed cost of FDI, $f_m$. Furthermore, relative export sales are lower in sectors with higher dispersion in firm domestic sales, i.e. those with lower $\kappa$ or greater $\sigma$.

29 To see this, use $y(\phi) = cp = p^{1-\sigma} \beta E \sigma^{-1}$ and substitute for $p$ using (7) and $\left( \frac{\sigma}{\sigma} \right)^{\sigma^{-1}} \beta E = \sigma A$. Using (12), we can express sales in terms of the productivity cutoff $\phi^*_d$. 

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productivity (φ). Using (26)-(27), we can express relative firm-level sales via FDI and exports as follows:

\[
\frac{t_m}{t_x} = \tau^{\sigma-1} \frac{H_m^{-(\sigma-1)}}{H_x^{-(\sigma-1)}}.
\]  

(28)

According to equation (28), relative sales via FDI increase in the relative scope \(g_m/g_x\) as defined in equation (20) and discussed in Proposition 3. Again, production technology governed by the parameter \(\alpha\) plays an important role. As shown above, the relative share of FDI products decreases in the flexibility of production (smaller values of \(\alpha\)). The higher is the flexibility of production, the lower is the cost differential among varieties within the firm and, hence, the smaller is the share of products sold via FDI. In turn, relative FDI sales according to equation (28) also decrease in production flexibility. We summarize these results in the following proposition.30

Proposition 5 The share of a firm’s FDI sales relative to its export sales increases in the relative scope for FDI \((g_m/g_x)\). For FDI firms (with productivity \(\phi \geq \phi^*_m\)), the relative share of FDI sales

i) decreases in the flexibility of production (smaller \(\alpha\)): \(\partial(t_m/t_x)/\partial\alpha > 0\),

ii) is constant in \(\phi\), increases in \(f_x\), \(\tau\), and decreases in \(f_m\).

Proof. See the proof to Proposition 3. Again, this result also holds in general equilibrium. ■

Next, we use (25)-(26) to derive the expression for export sales relative to domestic sales:

\[
\frac{t_x}{t_d} = \tau^{1-\sigma} \frac{H_x^{-(\sigma-1)}}{H_d^{-(\sigma-1)}}.
\]  

(29)

Similarly to our discussion regarding relative product scope in Proposition 4, the effect of technology depends on whether the firm is a multinational or not. For firms that conduct FDI, the share of export relative to domestic sales increases in the flexibility of production (smaller \(\alpha\)). The opposite is true for firms that only export. As mentioned above, a greater flexibility increases product scope and hence decreases the sales share of the most productive varieties. Since the most productive varieties are sold via FDI (exports) in multinational (exporting-only) firms, we derive differential effects for the two types of firms.

Proposition 6 The flexibility of production affects relative exports in multinational and exporting-only firms differently.

30Note that analogous results hold when comparing FDI sales to domestic sales \((t_m/t_d)\).
i) For multinational firms (with productivity $\phi \geq \phi^*_m$), the share of export sales in domestic sales increases in flexibility (smaller $\alpha$): $\frac{\partial (t_x/t_d)}{\partial \alpha} < 0$.

ii) For exporting-only firms (with productivity $\phi^*_x \leq \phi < \phi^*_m$), the share of export sales in domestic sales decreases in flexibility (smaller $\alpha$): $\frac{\partial (t_x/t_d)}{\partial \alpha} > 0$.

**Proof.** Relative exports sales $t_x/t_d$ increase in relative export scope $G_x/G_d$. According to equations (21) and (22), the technology parameter $\alpha$ affects the relative scope of multinationals and exporters differently (see the proof to Proposition 4). Again, this result also holds in general equilibrium. ■

### 3.4 Productivities at the plant level

In this subsection, we compare productivities of parent and affiliate plants of multinational multi-product firms. To do so, we make use again of the efficiency indices defined in equation (6) and used in equations (25)-(27). The respective productivities of the parent ($H_d(\phi)$) and the affiliate plant ($H_m(\phi)$) of a firm with core productivity $\phi$ are given by:

$$H_d(\phi) \equiv \left( \sum_{g=1}^{g_d} g \alpha^{\sigma-1} \right)^{-\frac{1}{\sigma-1}}$$

and

$$H_m(\phi) \equiv \left( \sum_{g=1}^{g_m} g \alpha^{\sigma-1} \right)^{-\frac{1}{\sigma-1}}. \quad (30)$$

Note again that firms face diseconomies of scope such that plant efficiency decreases in the number of produced varieties. Since we determine product scope at the plant level endogenously, our framework provides a rationale for differences in plant-level productivities, which are not present in standard models with single-product firms. Considering the ratio between the efficiency indices of foreign and domestic production ($H_m/H_d$) allows us to analyse relative productivity between the affiliate and the parent company. This ratio is equal to one in a framework with single-product firms (Helpman et al. (2004)), whereas it is endogenous and larger than one in our case, i.e. $\frac{H_m}{H_d \text{MPF}} > \frac{H_m}{H_d \text{SPF}} = 1$.\(^{31}\)

In consequence, any shock that affects relative product scope (e.g., globalization or a change in technology) will affect the relative productivity between affiliate and parent plants in our framework. This is an important novel implication of our model. According to our analysis above, core varieties are sold via FDI. Hence, our model suggests that affiliates are more productive than parent plants.

\(^{31}\)In our framework, it would be equal to one only in the hypothetical case where $g_m = g_d$, i.e. where all varieties that are sold in the domestic market are also sold abroad via FDI. This case, however, is ruled out by the existence of fixed costs.
which produce a comparatively larger range of domestic products.\footnote{Note that this result is derived in a setting where firms seek foreign market access and is, therefore, only valid for horizontal but not for vertical FDI. Eckel and Irlacher (2017) analyze vertical FDI in a setting of multi-product firms where marginal varieties with low productivities are relocated to save on factor costs. In such a setting, the productivity ranking between domestic and foreign plants is different to the one in our framework.}

We can use our framework to analyze the change in the relative efficiency between affiliates and parents, $H_m/H_d$, in response to changes in underlying parameter values. According to equations (30), relative efficiency decreases in the share of FDI products ($g_m/g_d$) defined in equation (19). Hence, we can directly use previous insights regarding relative product scope in Proposition 3 to determine how given changes in cost parameters or technology affect the relative productivity of plants. We summarize our findings in the following proposition.

**Proposition 7**  
*In a setting with horizontal FDI and multi-product firms, affiliates are more efficient in production than parent firms, i.e. $H_m/H_d > 1$. Any shock that decreases the relative scope of FDI products, $g_m/g_d$, increases the relative productivity advantage of the foreign affiliate.*

4 Conclusion

In this paper, we analyze the international expansion strategies of multi-product firms. While the most productive firms choose to become multinationals, FDI is not the optimal mode of serving foreign consumers for each variety within a firm. Firms that operate with a flexible manufacturing technology open new affiliates for the production of their core varieties (i.e., the varieties with the highest productivity) and, hence, the largest sales. Exporting is chosen as the optimal mode for varieties with an intermediate productivity. This way, our model is able to rationalize the empirical fact that the most productive firms typically rely simultaneously on both FDI and exporting. After having determined the conditions for the endogenous export versus FDI decision at the product-level, we derive a range of comparative statics results with respect to both changes in technology as well as globalization. This is important, since understanding the export versus FDI decision at the product-level is crucial for productivity at the plant-level. Our model suggests that any shock that affects production decisions at the product level also affects the relative productivity between the parent firm and its affiliate. These shocks also determine where the profits of the most profitable core varieties are recorded: at home in case of exporting, or abroad in case of FDI.
It should be interesting to further investigate the market access strategies of multi-product firms empirically using suitable data at the firm-product-destination level. We provide testable predictions regarding the productivity effects of product reallocations within the boundaries of the firm. Moreover, it would be interesting to estimate the role of production technology on the export/FDI mix in multinational firms along the lines we describe.
References


A Theoretical Appendix

A.1 Total operating profits

Total operating profits from selling an optimal number of products $G_{id}$, $G_{ix}$ and $G_{im}$ at optimal prices $p_{ijg}$ domestically, and via exports and FDI, respectively, are

$$\pi_{id}(\phi) = \max_{G_{id}} \sum_{g_{id}=1}^{G_{id}} \left[ \max_{\{p_{iig}\}_{g_{id}=1}^{G_{id}}} \left( p_{iig} - \frac{w_i}{\phi/h(g)} \right) \left( \frac{p_{iig}}{P_i} \right)^{-\sigma} \frac{\beta E_i}{P_i} \right],$$

$$\pi_{ix}(\phi) = \max_{G_{ix}} \sum_{g_{ix}=1}^{G_{ix}} \left[ \max_{\{p_{ijg}\}_{g_{ix}=1}^{G_{ix}}} \left( p_{ijg} - \tau_{ij} \frac{w_i}{\phi/h(g)} \right) \left( \frac{p_{ijg}}{P_j} \right)^{-\sigma} \frac{\beta E_j}{P_j} \right],$$

$$\pi_{im}(\phi) = \max_{G_{im}} \sum_{g_{im}=1}^{G_{im}} \left[ \max_{\{p_{ijg}\}_{g_{im}=1}^{G_{im}}} \left( p_{ijg} - \frac{w_j}{\phi/h(g)} \right) \left( \frac{p_{ijg}}{P_j} \right)^{-\sigma} \frac{\beta E_j}{P_j} \right].$$

A.2 General equilibrium

Due to free entry, expected profits are zero in equilibrium. That is, expected operating profits of a potential entrant are equal to market entry costs, given by $f_e$:

$$f_e = \int_{\phi_d^*}^{\infty} \Pi_d(\phi, g) dF(\phi) + \int_{\phi_x^*}^{\phi_d^*} \Pi_x(\phi, g) dF(\phi) + \int_{\phi_m^*}^{\infty} \Pi_m(\phi, g) dF(\phi), \quad (31)$$

where

$$\Pi_d(\phi, g) = \sum_{g=1}^{g_d} \pi_d(\phi, g), \quad \Pi_x(\phi, g) = \sum_{g=1}^{g_x} \pi_x(\phi, g), \quad \Pi_m(\phi, g) = \sum_{g=1}^{g_m} \pi_m(\phi, g) + \sum_{g=g_m+1}^{g_x} \pi_x(\phi, g).$$

Using (9)-(11) to substitute for $\pi_d(\phi, g)$, $\pi_x(\phi, g)$ and $\pi_m(\phi, g)$ and (15)-(17) to substitute for $g_d$, $g_x$ and $g_m$, the free-entry condition (31) and the zero-cutoff-profit conditions (12)-(14) provide implicit solutions for the cutoff productivities $\phi_d^*$, $\phi_x^*$ and $\phi_m^*$ and the demand level $A \equiv \frac{1}{\sigma} \left( \frac{\sigma-1}{\sigma} \right)^{-1} B$, which depends on the range of available varieties via the price index $P$ (see equation (3)).

Averaging first over the $g$-th variety produced by all firms and then summing over all $g$ (compare
Mayer et al. (2014)), we can write:

\[ f_e = \sum_{g=1}^{\infty} \int_{\phi_d^*}^{\infty} (A\phi_d^* - f_d)dF(\phi) + \sum_{g=1}^{\infty} \int_{\phi_x^*}^{\infty} (A\phi_x^* - f_x)dF(\phi) + \sum_{g=1}^{\infty} \int_{\phi_m^*}^{\infty} (A\phi_m^* - f_m)dF(\phi). \] (32)

To pin down \( \phi_d^* \) (and, hence, \( \phi_x^* \) and \( \phi_m^* \)), consider the free entry condition (32) and use the zero-cutoff-profit conditions (12)-(14) to substitute for \( A = \frac{f_d}{(\phi_d^*)^{\sigma-1}}, A\tau = \frac{f_x}{(\phi_x^*)^{\sigma-1}} \) and \( A(1-\tau) = \frac{f_m-f_x}{(\phi_m^*)^{\sigma-1}} \). We further assume that firm productivities are drawn from a Pareto distribution following Helpman et al. (2004) with a scale parameter \( b = 1 \) and shape parameter \( \kappa > 1/\alpha \), such that \( F(\phi) = 1 - \phi^{-\kappa} \) with \( dF(\phi) = \kappa\phi^{-\kappa-1}. \)\(^{34}\) Solving for the integrals and simplifying, we get:

\[ \phi_d^* = (B\Omega)^{\frac{1}{\kappa}}, \] (33)

where

\[ B \equiv \frac{\kappa}{f_e} \left( \frac{1}{\kappa - \sigma + 1} - \frac{1}{\kappa} \right) f_d^{\frac{\kappa}{\kappa - \sigma - 1}} \left[ f_d^{\frac{1}{\kappa - \sigma - 1}} + f_x^{\frac{\kappa}{\sigma - 1}} \tau^{\kappa} + (f_m - f_x)^{\frac{\kappa}{\sigma - 1}} (1 - \tau^{1-\sigma})^{\frac{\kappa}{\sigma - 1}} \right] \text{ and} \]
\[ \Omega \equiv \sum_{g=1}^{\infty} g^{-\alpha\kappa}. \]

Note that \( \sum_{g=1}^{\infty} g^{-\alpha\kappa} \) converges due to the assumption that \( \alpha\kappa > 1 \).

**Result.** The cutoff productivity \( \phi_d^* \) increases in response to i) greater production flexibility (smaller values of \( \alpha \)), and ii) a reduction in trade costs, \( f_x \) and \( \tau \). In turn, domestic product scope, \( g_d \), decreases.

**Proof.** Regarding i), \( \frac{\partial \phi_d^*}{\partial \alpha} < 0 \) directly follows from (33). Regarding ii), note that \( \frac{\partial \phi_d^*}{\partial f_x} = \frac{1}{\kappa} (B\Omega)^{\frac{1}{\kappa} - 1} \Omega \frac{\partial B}{\partial f_x} \) and \( \frac{\partial \phi_d^*}{\partial \tau} = \frac{1}{\kappa} (B\Omega)^{\frac{1}{\kappa} - 1} \Omega \frac{\partial B}{\partial \tau} \), where

\[ \frac{\partial B}{\partial f_x} = \frac{f_d^\kappa}{f_e} \left[ -f_x^{\frac{\kappa}{\sigma - 1}} \tau^{\kappa} + (f_m - f_x)^{\frac{\kappa}{\sigma - 1}} (1 - \tau^{1-\sigma})^{\frac{\kappa}{\sigma - 1}} \right] < 0 \]

The bounds of the three integrals correspond to the values of \( \phi \) such that \( E(\pi_d(g)) \geq 0 \) (first integral, lower bound), \( E(\pi_x(g)) \geq 0 \) and \( E(\pi_m(g)) \geq E(\pi_m(g)) \) (second integral, lower and upper bound), and \( E(\pi_m(g)) \geq E(\pi_x(g)) \) (third integral, lower bound).

\(^{34}\)We assume \( \kappa > \sigma + 1 \). This ensures that the distribution of productivity draws has a finite variance. (A Pareto random variable has a finite variance if and only if \( \kappa > 2 \).)
and

\[
\frac{\partial B}{\partial \tau} = \frac{f_d^\kappa}{f_e} \frac{(\sigma - 1) \kappa}{\kappa - \sigma + 1} \left[ -\frac{\sigma - 1 - \kappa}{\sigma - 1 - \kappa} \tau^{\kappa - 1} + (f_m - f_x) \frac{\sigma - 1 - \kappa}{\sigma - 1 - \kappa} (1 - \tau^{1 - \sigma}) \frac{\kappa - 1}{\sigma - 1} \tau^{-\sigma} \right] < 0
\]

since \( f_m > \tau^{\sigma - 1} f_x \). Furthermore, \( \frac{\partial g_d}{\partial \phi_d} < 0 \) according to (18). \( \blacksquare \)