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Fiscal illusion and the shadow economy: Two sides of the same coin?

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Working Paper No. 1321
December 2013

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Fiscal illusion and the shadow economy: Two sides of the same coin?

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Abstract: This paper presents an empirical analysis of the relationship between fiscal illusion and the shadow economy for 104 countries over the period 1989–2009. We argue that both unobservable phenomena are closely linked to each other, as the creation of a fiscal illusion may be helpful if governments want to control shadow economic activities. Using a MIMIC model with two latent variables we confirm previous findings on the driving forces of the shadow economy and identify the main determinants and indicators of fiscal. Most importantly, we find that fiscal illusion negatively affects the shadow economy: Concealing the real tax burden through fiscal illusion potentially contributes to the government's efforts to repress shadow economic activities.

JEL Classification: *O17, K42, O54, N16*

Keywords: Fiscal illusion; shadow economy; MIMIC model; latent variables, tax burden, tax complexity.

1. Introduction

This paper expands previous research on the shadow economy and fiscal illusion. Both, the shadow economy, i.e. the production and distribution of goods and services that is concealed from the government, and fiscal illusion, i.e. the systematic delusion of key fiscal parameters by taxpayers, are two important – to our opinion – closely linked economic phenomena. On the one hand, the more effective the government is creating a fiscal illusion, the more likely it is that voters underestimate the actual or true tax burden of government activities. This potentially affects the size and development of the shadow economy, as the tax burden is often found to be its most important determinant. Hence, the systematic misperception of the true tax burden should reduce people's incentives to work in the shadow economy, as they do not feel depleted that much by public spending. On the other hand, the existence of a large shadow economy potentially contributes to the creation of fiscal illusion as well. In countries with a large shadow economy, weak institutions and an environment of mistrust towards government policies may make only the instrument of fiscal illusion available to the government to reduce the (perceived) pressure of taxation and thus the shadow economy. Hence, a sizable shadow economy can go hand in hand with a high level of fiscal illusion.

Although both phenomena are not observable, they leave traces such as the frequency of cash transactions and the complexity of the tax system that can be used to study their relationship. For the first time we analyze the interaction between the shadow economy and fiscal illusion using a multiple indicator multiple causes (MIMIC) model with two latent variables. Selecting appropriate causes and indicator of these two unobservable phenomena we investigate the driving forces behind the shadow economy and fiscal illusion. Differently to previous studies applying a MIMIC model, we do not focus on the

measurement of either latent variable. Rather we apply the MIMIC model to explore the mutual interactions between the shadow economy and fiscal illusion. We hypothesize that the better a government is able to "create" a fiscal illusion, the smaller the shadow economy is, all other things being equal. Hence, the government may use fiscal illusion as an additional tool to control shadow economic activities. A second contribution of this paper is to join two strands of the literature, i.e., the literature on fiscal illusion and the literature on the shadow economy.

The paper is organized as follows: Section 2 discusses some theoretical considerations about the potential relationship between fiscal illusion and the shadow economy. In Section 3, we present the empirical analysis modeling the shadow economy and fiscal illusion simultaneously in a MIMIC model for the first time. Section 4 briefly summarizes the most important findings and concludes.

2. Fiscal Illusion and the Shadow Economy

The traditional view on the concept of fiscal illusion is the systematic misperception of key fiscal parameters (taxes) by taxpayers, distorting fiscal choices.[1] Mill (1848 [1994], p. 237) already discussed the perception of different taxes: ''If all taxes were direct, taxation would be much more perceived than at present, and there would be a security, which now there is not, for economy in the public expenditure.'' Mill's seminal observation indicates that one important nature of fiscal illusion is political illusion. It occurs when politicians use fiscal instruments to deceive taxpayers making them feel paying less than they are

[1] The paper focuses on the relationship between fiscal illusion and the shadow economy and does for this reason discuss the literature only briefly. A comprehensive literature review on fiscal illusion is presented in Dell'Anno and Mourão (2012). The empirical literature on fiscal illusion is surveyed in Dollery and Worthington (1996). Schneider and Enste (2000) present an excellent survey about the shadow economy, also including different measurement methodologies, which we do not discuss here.

actually contributing to government programs (Fasiani 1941). In this sense, taxpayers potentially attribute more value to public expenditures than they are worth, which in the end leads to a public sector of excessive size (Oates 1988).

To disguise taxpayers, politicians have several options. Firstly, designing a tax system more complex makes it more difficult for taxpayers to understand its significant elements. As a consequence they very likely underestimate their effective tax burden, allowing the government to increase public expenditures without the full perception of taxpayers (Wagner 1976; Cullis and Jones 1987). Fiscal illusion is also created if governments finance expenditures by debt rather than by tax revenues. According to the Ricardian equivalence theorem people would be indifferent between debt and tax financing if they had rational expectations. Since they do not, they are subjected to a fiscal illusion or more precisely debt illusion, underestimating future tax liabilities in the form of current public debt. In other words, current taxation generates higher levels of perception of the true burden than public indebtedness. This distortion leads to a systematic underestimation of public expenditures and the cost of government programs. Fiscal illusion is thus the systematic distortion of taxpayer's perceptions by the government.

In contrast, the shadow economy is often defined as all economic activity – and income earned from it – that circumvent government regulation, taxation, or observation. Hence, shadow economic activities include unreported income from otherwise official trade in goods and services, i.e., all economic activities that would generally be taxable were they reported to governmental (tax) authorities are part of the shadow economy. This broad definition of the shadow economy is however difficult to implement empirically. To make it applicable for the empirical analysis presented in section 3, we

introduce the following more narrow definition: the shadow economy comprises all market-based, lawful production or trade of goods and services deliberately concealed from public authorities in order to evade either payment of income, value added or other taxes, or social security contributions; to get around certain labor market standards, such as minimum wages, maximum working hours, or safety standards; or to avoid compliance with administrative procedures, such as filling out paperwork. This definition does not include illegal economic activities, such as burglary, robbery, or drug dealing.

The discussion of both phenomena in the preceding two paragraphs suggests that fiscal illusion and the shadow economy are interrelated phenomena and may be two sides of the same coin. On the one hand, an economy with a large shadow sector reduces the quality of institutions and is potentially characterized by low attitudes towards the state. Hence, policymakers are probably keen to apply several strategies limiting the size of the shadow economy. Standard policy instruments often recommended by economist are – in addition to tax reforms reducing the tax and regulatory burden – to increase the effectiveness of tax auditing, the enforcement of tax rules and regulations, or the punishment of shadow workers or tax evaders. An alternative way for politicians to deplete the shadow economy may is to systematically distort citizen's true tax burden. Assuming that policymakers explore all available policy options, a higher shadow economy can potentially be an incentive for policymakers to adopt strategies to hide the true tax burden to taxpayers. In this way they can avoid a further increase or even induce a reduction of tax evasion and the shadow economy. Hence, a higher shadow economy may lead to a higher level of fiscal illusion, all other things being equal.

If the government successfully creates the illusion of a lower tax burden, individuals do have fewer incentives to escape into the shadow economy or to evade taxes. As a

consequence, the shadow economy should be smaller in the presence of a high level of fiscal illusion. Alternatively, however, it might be possible that the existence of the shadow economy is just an indication of the government's failure to create a fiscal illusion. Because citizens correctly perceive their true tax burden, they realize contributing too much to government programs. By escaping into the shadow economy or evading taxes, they can reduce their effective tax burden to a level that matches the value they attribute to public expenditures programs. The next section presents a structural equation model, which allows us to empirically investigate this mutual relationship between fiscal illusion and the shadow economy.

3. The Empirical Analysis

3.1 The SEM and MIMIC approaches

Structural Equation Models (SEM) are based on statistical relationships among latent (i.e. unobservable) and manifest (i.e. observable) variables to simultaneously estimate relationships between multiple independent, dependent and latent variables. Combining factor analysis and the multivariate regression model, SEM integrate two important aspects of economic analysis: (1) variable measurability and observability and (2) the identification of their causal relationships. In this paper, a special type of a SEM is employed, i.e., a MIMIC model with two latent variables, to study the nexus between fiscal illusion and the shadow economy.

A MIMIC model has two parts: a measurement model and a structural model. The measurement model specifies the relationships between latent variables (shadow economy and fiscal illusion) and their indicators. In matrix notation, it is given by:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_{24} & \lambda_{25} & \lambda_{26} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} \tag{1}$$

where the latent variables (f_1 and f_2) determine linearly, subject to disturbances ε, a set of six endogenous indicators (y). Each of these latent variables has three observable indicators. The covariance matrix of the measurement errors, ε, is given by the matrix Θ^{ε}.[2]

The structural equation model linearly determines the latent variables f_1 and f_2 by a set of eight exogenous causes (x). Because the structural equation model only partially explains the latent variables, the structural disturbance error terms ζ_1 and ζ_2 represent the unexplained components. We assume B to be a (2×8) matrix of structural coefficients describing the "causal" relationships between the latent variables f_1 and f_2 and their causes. In matrix notation, it is given as:

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0 & \eta_{12} \\ \eta_{21} & 0 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} & \beta_{15} & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_{24} & \beta_{25} & \beta_{26} & \beta_{27} & \beta_{28} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} + \begin{bmatrix} \zeta_1 \\ \zeta_1 \end{bmatrix} \tag{2}$$

[2] In the standard MIMIC model (Jöreskog and Goldberger 1975), the measurement errors are assumed to be independent of each other, but this restriction can be relaxed (Stapleton 1978). In this paper, several covariances between indicators are relaxed since they are empirically and theoretically plausible. Figure 1 shows some of these estimated covariances.

Without loss of generality, all variables are considered to carry zero expectations, i.e., $\left[E(f)=E(x)=E(y)=0\right]$, and the variances of the structural disturbance error terms ζ_1 and ζ_2 are abbreviated by a non-diagonal matrix Ψ. The MIMIC model assumes that $E(\zeta)=E(\varepsilon)=0$; the error terms do not correlate with the causes $\left[E(x\zeta)=0\right]$; the error terms in the measurement model do not correlate either with the causes $\left[E(x\varepsilon')=0\right]$ or with the latent variables $\left[E(f\varepsilon')=0\right]$; and, finally, the measurement errors do not correlate with structural disturbances $\left[E(\varepsilon\zeta)=0\right]$.

There exist several equivalent ways to represent a SEM. One of the simplest is the RAM (*Reticular Action Model*) formulation of McArdle (1980) and McArdle and McDonald (1984). It allows to fully specifying a path model by linear equations between manifest and latent variables using three matrixes only instead of several model matrices as done by LISREL. In particular, the RAM formulation considers a vector *v* containing the observable indicator variables, the observable causal variables and the latent variables, and a vector *u*, including the observable causal variables, the measurement errors, and the structural disturbances. The two sets of variables, i.e., vectors, are linked by equation (3) as follows:

$$v = Av + u\,, \tag{3}$$

where A is a matrix including the structural and measurement coefficients. The covariance matrix of *u* is $P=E(uu')$.[3] Furthermore, $W=E(vv')$ denotes the covariances of the observables, computed directly from the sample. Assuming that $I-A$ is non-

[3] Appendix B presents estimated the elements of the A and P matrices for our favourite specification (MIMIC 8-2-6a). To facilitate understanding of the MIMIC model presented in this paper for LISREL-users, we adopt LISREL symbols in the RAM formulation.

singular, equation (3) can be rewritten as $v = (I - A)^{-1} u$ and $W = (1 - A)^{-1} P (1 - A)^{-1'}$. Let $\sum = E(mm')$ be the estimated covariance matrix of the observable variables and *J* a "filter matrix" which carries v into $m = Jv$, we get: $\sum = E(mm') = JE(vv')J' = JWJ' = J(1 - A)^{-1} P (1 - A)^{-1'} J'$. Assuming multivariate normality, the maximum-likelihood estimates of the parameters in A and P are calculated by minimizing the discrepancy between *W*, and the covariance matrix $\sum$ implied by the model:

$$F_{ML} = \ln|\sum| + tr(W \sum^{-1}) - \ln|W| - n, \quad (4)$$

where |.| indicates the determinant of a matrix, *tr* indicates its trace, *n* is the sum of the number of observable endogenous indicators (*y*) and observable exogenous causes (*x*). The necessary condition for identification is that the number of structural parameters should be equal to the number of reduced-form parameters. An observation of the reduced-form parameters shows that unique solutions to the measurement and structural parameters λ and β cannot be obtained from the reduced-form model. This occurs because altering the scale of either f_1 and f_2 yields an infinite number of solutions for λ and β from the same reduced-form solution. The inability to obtain unique solutions for λ and β causes an identification problem that can be solved by (i) constraining one of the paths from the latent variable to one of its indicator variables, or by (ii) fixing the variances of the structural disturbance error terms, ψ_{11} and ψ_{22} to 1. In this paper we

consider the latter alternative to identify the model more appropriate as we do not aim to use the MIMIC model estimates to assess the size of the unobservable variables.[4]

3.2 Observable structural causes and indicators of both latent variables

An extensive literature exists on the empirical analysis of the shadow economy and fiscal illusion. The previous section has made clear that the rationale behind the selection of the observable variables is a key issue for the MIMC approach. Duncan (1975) points out that the meaning of the latent variables, and hence the reliability of the estimates, depends on how comprehensively the causal and indicator variables correspond to the intended content of the latent variables. To define the latent variable shadow economy as precise as possible, six potential causes and three indicators are chosen. Given data availability, the structural model investigates the relationships between the shadow economy (f_1) and the following variables[5]:

X_1, Personal income tax: The higher (lower) the individual income tax burden, the larger (smaller) the shadow economy, ceteris paribus; i.e., $\beta_{11}>0$.

X_2, Corporate income tax: The higher (lower) the corporate income tax rate, the larger (smaller) the shadow economy, ceteris paribus; i.e., $\beta_{21}>0$.

X_3, Unemployment rate: The higher (lower) the unemployment rate, the more (less) time and incentives people have to work in the shadow economy, ceteris paribus; i.e., $\beta_{31}>0$.

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[4] The first alternative is superior when the model is used to estimate the size of latent variables as it anchors the meaning of latent variable to the dimension of the reference indicator. See Dell'Anno (2007) for details.

[5] There is an intensive discussion, why the variables X_1, X_2, X_4 and X_5 are key factors or driving forces for shadow economy activities. Compare for example Schneider and Enste (2000), Schneider (2005) and Feld and Schneider (2010).

X_4, Business freedom: A fourth important determinant may affecting the size of the shadow economy is the burden of regulation for business activities. To take the extent of business regulations into account, we employ the index of business freedom, as estimated by the Heritage Foundation. It seems reasonable to assume that the greater the business freedom (i.e. the higher the index score), the lower the size of the shadow economy. According to this view, we expect the estimated coefficient to be negative, i.e., $\beta_4<0$.

X_5, Tax burden: Tax revenues as percentage of GDP are used as measure of the overall tax burden in an economy, which potentially influences both, the extent of fiscal illusion and the size of the shadow economy. The higher the tax burden the stronger the incentives for individuals to operate in the shadow economy and for the government to delude the true burden through fiscal illusion. We thus use the overall tax burden as potential cause for both the shadow economy and fiscal illusion and expect a higher (lower) overall tax burden to provoke more (less) shadow economic activities, ceteris paribus, i.e., $\beta_{51}>0$.

For fiscal illusion, four main structural causes, enhancing the efficacy of fiscal illusion, and three main categories of policies, capable of distorting taxpayers' perceptions of their true tax burden, are selected. As argued above, we believe that the tax burden can be seen as a proxy for policymakers' needs to reduce the perception of tax pressure. A higher (effective) tax burden encourages the government to adopt tax policies aimed at increasing fiscal illusion. Thus, the expected correlation between the overall tax burden and fiscal illusion is positive, i.e., $\beta_{52}>0$. Further important determinants of fiscal illusion are:

X_6, Self-employment: This variable is considered as potential cause for both fiscal illusion and the shadow economy. The shadow economy literature presents unambiguous evidence that self-employed have much more possibilities to work in the shadow economy, hence the higher the self-employment ratio is, the larger the shadow economy should be, ceteris paribus, i.e. $\beta_{61}>0$. [6]

Concerning fiscal illusion, a higher ratio of self-employed to the totally employed population can increase the policymaker's needs to conceal the tax burden. Fasiani (1941) already argued that a higher self-employment ratio requires a higher degree or more "active" tax compliance as the system of withholding income tax is rather partial for self-employed. Hence, we expect a higher self-employment ratio to increase the level of fiscal illusion because it incentivizes policymakers to distort the perception of the tax burden, i.e. $\beta_{62}>0$.

X_7, Top income tax rate: We assume that a higher (statutory) top income tax rate encourages a government to adopt tax policies aimed at creating a fiscal illusion, because a highly visible statutory top income tax rate very likely produces perceptions of a burdensome tax regime, which result in high electoral cost for the government. Thus, the expected correlation between the top (statutory) income tax rate and fiscal illusion is positive, i.e. $\beta_{72}>0$.

X_8, Secondary school enrolment: The fourth potential cause of fiscal illusion takes into account the ability of a society to correctly evaluate the beneficiaries of both tax reforms and public expenditure programs. Assuming that this ability depends on the average level of education, we take into account the secondary school enrolment rate,

[6] Compare e.g. the survey of Feld and Schneider (2010) and the references mentioned there.

i.e. the ratio of children enrolled in secondary education to the population of the official secondary education age. A more educated society makes it more difficult for policymakers to effectively implement fiscal illusion policies to distort taxpayers' perceptions. Thus, we expect a negative correlation between the secondary school enrolment rate and fiscal illusion, i.e. $\beta_{82}<0$.

Finally, we include a dummy variable for the OECD countries ($\mathbf{X_9}$) to verify whether structural differences between OECD countries and non-OECD countries exist. The measurement model includes three variables that are typically used in the literature as indicators of the shadow economy. In addition, three variables, representing the most common strategies to reduce citizens' perceptions of the true tax burdens, are employed as indicators of fiscal illusion. In particular, the measurement model links the following six indicators to the unobservable variables:[7]

$\mathbf{Y_1}$, Labor force participation: A low (high) labor force participation rate in the official economy may be seen as indication of prevalent (rare) shadow economic activities, ceteris paribus. We thus expect $\lambda_{11}<0$.

$\mathbf{Y_2}$, Growth rate of real GDP: The theoretical literature does not offer unambiguous guidance concerning the effects of the shadow economy on official economy and vice versa.[8] For instance, Kaufmann and Kaliberda (1996) show that countries experiencing a decline in official GDP were able to mitigate such a drop through growth of the shadow economy. On the contrary, Chong and Gradstein (2007) find a positive relationship between the shadow economy and official growth. Following the slight

[7] There is a vast literature on possible indicators of the shadow economy. Compare e.g. Schneider and Enste (2000), Feld and Schneider (2010), and Schneider et al. (2010).
[8] For an overview see Dell'Anno (2008).

prevalence of a positive correlation found in the empirical literature, we expect that the higher the official growth rate of real GDP is, the larger the shadow economy, ceteris paribus, i.e., $\lambda_{12}>0$.

Y_3, Currency ratio: It is commonly assumed that cash is used to pay for goods and services produced in the shadow economy as it leaves no trace compared to bank transfers or other tractable payment methods. We thus expect a positive sign for the coefficient of the currency ratio, ceteris paribus, i.e., $\lambda_{13} >0$.

Y_4, Public debt: A common strategy to create fiscal illusion is to increase public debt. The motivation behind this argument is that taxpayers are more likely to perceive the cost of public programs if they pay for them through current taxation than if tax liabilities are deferred through public-sector borrowing (Oates 1988). Hence, we expect a positive correlation between fiscal illusion and public debt, ceteris paribus, i.e., $\lambda_{24}>0$.

Y_5, Share of indirect taxation: The second variable included as indicator of fiscal illusion is the share of indirect taxation. According to the "Mill hypothesis", fiscal extraction through indirect taxation is underestimated compared to direct taxation because it is less visible to taxpayers. The "Mill' hypothesis" stressed by Schmölders (1960) and Buchanan (1967), represents one of the most common forms policymakers use to reduce the perceived sacrifice of taxpayers. In this sense, a positive coefficient is expected for the share of indirect taxation, ceteris paribus, i.e., $\lambda_{25}>0$.

Y_6, Tax complexity: Finally, we use the complexity of the tax system as indicator of fiscal illusion. The more complex and complicated a tax and revenue system, the more likely it is that taxpayers underestimate the tax burden and misperceive the true tax

liabilities, all other things being equal. Following Wagner (1976), we compute the Herfindahl index of a country's revenue system (H).[9] A higher value of this index means a less complex revenue system; the revenue-complexity hypothesis thus posits a negative coefficient for the Herfindahl index of tax complexity, i.e., $\lambda_{26} < 0$.

Figure 1 shows the path diagram of the final MIMIC model, including eight causes, two latent variables and six indicators. It has been estimated for an unbalanced panel of a cross-section of 104 countries over the period 1989 to 2009 (21 years). The list of countries included in the sample as well as the definitions and data sources are provided in Appendix A. As common, the observable variables in Figure 1 are represented by rectangles and the latent variables by ovals. An arrow represents the effect of one variable on the other and the parameters represent the coefficients to be estimated. The arrows linking indicators and causes among themselves indicate the estimated covariances among the structural and measurement errors in the MIMIC model.

To make the SEM approach suitable to the data set's panel structure, we transform the observable variables into deviations from the country mean over the sample period. This transformation meets the assumption that all variables have zero expectations, i.e. $\left[E(F) = E(x) = E(y) = 0\right]$, since the variables now have the same mean (zero) across countries. The deviations from the country mean are computed as follows:

$$x_{jit} = \left(x^{r}_{jit} - \sum_{t} x_{ji} \right); \; y_{jit} = \left(y^{r}_{jit} - \sum_{t} y_{ji} \right), \tag{5}$$

where superscript *r* denotes raw data; j = 1, 2,…, 14 indicates the observable causes and indicators variables; i = 1, 2,…, 104 denotes the number of countries; and t = 1989,…,

[9] We follow the literature and use different types of taxation to compute the Herfindahl index. See Appendix A for details.

2009 specifies the observation period. This approach makes it feasible to consider heterogeneity across cross-sectional units in the MIMIC model and is motivated by the relevance of country fixed effects in the model.

Figure 1: Path Diagram MIMIC 8-2-6a

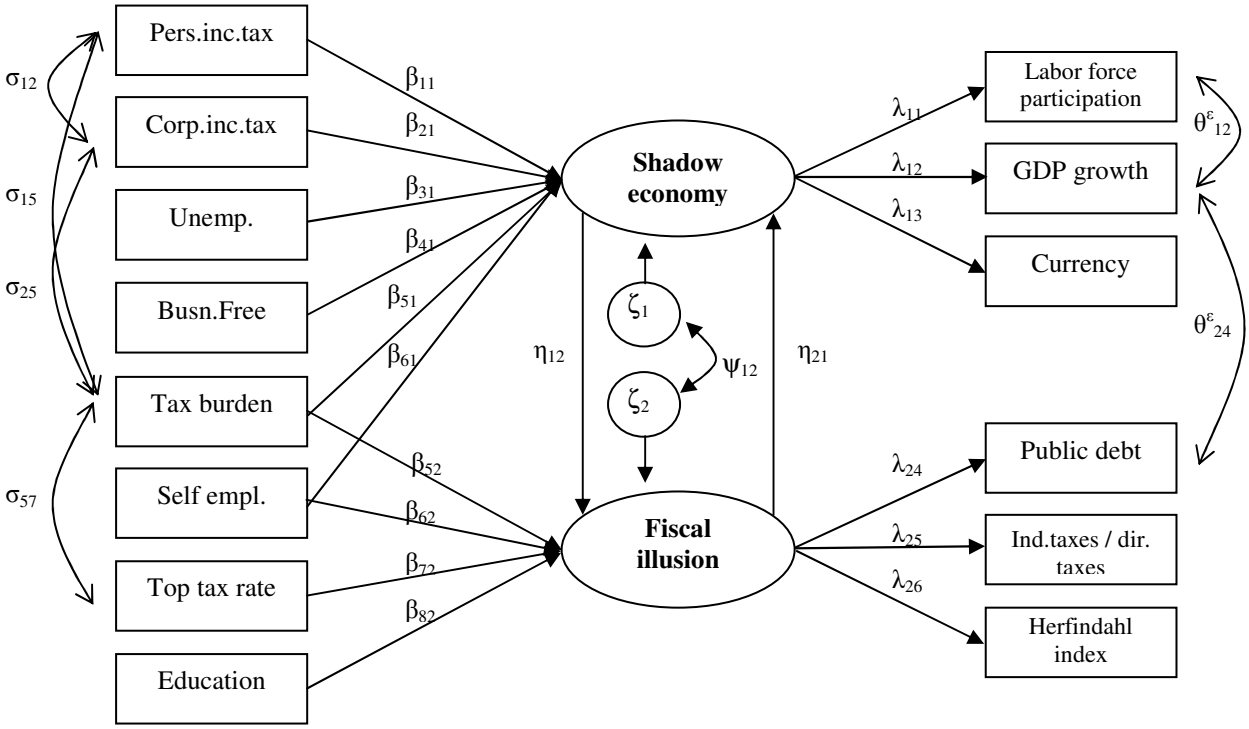

Table 1 shows the results of three MIMIC model specifications: MIMIC 8-2-6a is our favorite model; MIMIC 8-2-6b and MIMIC 6-2-6 are models that include a dummy for the OECD countries (x_9) to take into account unobserved differences between OECD and non-OECD countries.

Table 1: MIMIC models and parameter estimates

		MIMIC 8-2-6a		MIMIC 8-2-6b		MIMIC 6-2-6	
Causes		SE	FI	SE	FI	SE	FI
Personal income tax	β_{11}	0.002 (0.291)	-	0.006 (0.806)	-	-	-
Corporate income tax	β_{21}	0.045* (2.507)	-	-0.049* (-5.468)	-	0.023 (1.449)	-
Unemployment rate	β_{31}	-0.010* (-2.256)	-	0.019* (5.407)	-	0.082* (-3.738)	-
Business Freedom index	β_{41}	0.002 (1.618)	-	-	-	0.013* (3.066)	-
Tax burden	β_{51} ; β_{52}	0.009* (2.596)	0.015* (5.417)	-	-0.021* (-5.269)	-	-
Self-employment rate	β_{61} ; β_{62}	0.002 (0.934)	-0.003 (-1.265)	-0.004* (-2.680)	0.005* (2.175)	-	-
Top income tax rate	β_{72}	-	-0.002 (-1.593)	-	0.004* (2.421)	-	-0.054* (-4.097)
Secondary education	β_{82}	-	-0.002* (-2.289)	-	0.005* (4.287)	-	-0.019* (-3.292)
OECD dummy	β_{91} ; β_{92}	-	-	0.002 (0.174)	0.000 (0.017)	-0.002 (-0.054)	0.045 (0.628)
Indicators							
Labor force participation	λ_{11}	36.794 (0.509)	-	-7.982* (-794.0)	-	5.406* (2.766)	-
Growth rate of real GDP per capita	λ_{12}	4.774 (0.403)	-	0.214* (9.263)	-	-8.961* (-2.578)	-
Currency ratio	λ_{13}	-22.066 (-0.469)	-	0.046 (0.846)	-	-4.641 (-1.300)	-
Public debt in % of GDP	λ_{24}	-	0.156 (1.283)	-	33.801* (4.600)	-	-30.867* (-4.031)
Indirect taxes/ direct taxes	λ_{25}	-	-6.315* (-144.7)	-	4.538* (5.589)	-	0.016 (0.739)
Herfindahl index of tax revenue	λ_{26}	-	-0.001* (-3.869)	-	-0.073* (-4.79)	-	-0.003* (-3.302)
Interactions terms							
Shadow ec. → *fiscal illusion* (η_{12})	η_{12}	4.464 (0.508)		1.014* (278.740)		4.192* (2.711)	
Fiscal illusion → *shadow ec.* (η_{21})	η_{21}	-0.994* (-120.074)		-0.482* (-2.925)		-0.943* (-35.918)	
Goodness-of-fit statistics							
χ^2(p-value)		373.066 (0.00)		272.802 (0.00)		207.752 (0.00)	
Degrees of freedom		66		49		45	
RMSEA		0.0462		0.0457		0.0407	
P-value for test of close fit (RMSEA<0.05)		0.913		0.903		0.997	

Note: t-Statistics are given in parentheses. * means |t-statistic|>1.96. The degrees of freedom are determined using the expression 0.5(n)(n+1)–t, where n is the number of observable causes and indicators and t the number of free parameters. All models assume that the matrix of structural disturbances (ψ_{12}) is symmetric. In specification MIMIC 8-2-6b, the matrix of covariances among the causes (σ_{ij}) is set free, excluding the covariances among the dummy of OECD countries and the remaining causes.

Several criteria can be used to assess the fit of SEM models. The chi-square (χ^2) distribution for the goodness of fit is the classical test to evaluate differences between the observable data and the model prediction. A small χ^2 is typically a sign of a good model fit. This test is however sensitive to the sample size; for samples of more than 200 observations, as in our study, the test tends to reject the model even when the fit is adequate (Barrett 2007). In other words, large samples will increase the chance of observing p-values lower than 0.05. Due to this drawback of the χ^2-test, alternative fit statistics have been developed. One of the most frequently used statistics is the Root Mean Square Error of Approximation (RMSEA) proposed by Steiger and Lind (1980). The RMSEA incorporates a penalty function for poor model parsimony and thus becomes sensitive to the number of parameters estimated and relatively insensitive to the sample size (Brown 2006). A rule of thumb is that RMSEA values less than 0.08 indicate an adequate fit and values below 0.05 suggest an excellent fit (Browne and Cudeck 1992). P-values higher than 0.10 for the test of close fit based on the RMSEA indicate a good fit; the fit is inadequate if the p-value is below 0.05. The RMSEA p-values of all three models estimated and shown in Table 1 reveal a good fit.

However, in our analysis we encounter frequent cases of indefinite matrix problems, which limits our options to specify alternative MIMIC models and makes the estimates less robust across different model specifications. Monte Carlo studies though demonstrate (see e.g. Anderson and Gerbing 1984; Boomsma 1982, 1985) that problems of non-positive definite matrices arise frequently when data provides relatively little information such as few observable indicator variables, small factor loadings or a high number of missing values (Bollen and Long 1993). Given that both specifications MIMIC 8-2-6b

and MIMIC 6-2-6 do not have positive (semi-)definite matrices of structural errors, we consider the specification MIMIC 8-2-6a as the most reliable model.

In this specification, the estimated coefficient between fiscal illusion and the shadow economy is negatively significant, while the coefficient of the effect from the shadow economy to fiscal illusion is insignificant. That is, if fiscal illusion increases the shadow economy reduces, all other things being equal, while a variation in the size of the shadow economy does not have a feedback effect on fiscal illusion. Although the two alternative MIMIC model specifications, i.e., MIMIC 8-2-6b and MIMIC 6-2-6, point to a significantly positive relationship between the shadow economy and fiscal illusion, we are cautious to draw conclusions from this result due to the econometric imperfections as outlined in the previous paragraph.

The results of the MIMIC model concerning the size and development of the shadow economy clearly show that the overall tax burden and the corporate income tax have the expected positive sign and are statistically significant. The unemployment rate has a negative, statistically significant sign in the favorite specification, which may be seen as a surprising finding because it is usually argued that the higher the unemployment rate the more labor is supplied in the shadow economy. People simply try to earn additional income to compensate utility losses due to unemployment. However, Tanzi (1999) as well as Buehn and Schneider (2012) argue that the effect of unemployment on the shadow economy is ambiguous, i.e., both a positive and negative sign may be observed in an empirical analysis. Buehn and Schneider's line of reasoning is as follows: income losses due to unemployment reduce demand in both the shadow and official economies. A substitution of official demand for goods and services for unofficial demand takes place as unemployed workers turn to the shadow economy – where cheaper goods and services

make it easier to countervail utility losses. This behavior may stimulate additional demand in the shadow economy. If the income effect exceeds the substitution effect, a negative relationship develops. Likewise, if the substitution effect exceeds the income effect, the relationship is positive. Moreover, the ambiguous effect of unemployment on the shadow economy may not only be due to the countervailing forces of the income and substitution effect but a consequence of a supply side effect when the unemployed search for and take up jobs in the shadow economy. While the shadow economy in this case clearly increases, the behavior of the unemployment rate depends on whether informal worker are considered unemployed in the official statistics or not. In the case informal workers are considered unemployed and part of the official unemployment statistics, the unemployment rate does not change. However, if informal workers are not considered unemployed unemployment decreases and one would observe a negative relationship between the shadow economy and unemployment. Hence, the relationship between unemployment and the shadow economy is less clear and is – as explained above – theoretically ambiguous. It is thus not unlikely to observe a negative coefficient in an empirical analysis. The remaining causes, personal income tax rate, the business freedom index, and the self-employment rate have positive signs but are not statistically significant.

Concerning the indicators of the shadow economy, the results are less conclusive. The labor force participation rate and the growth rate of official real GDP per capita are not statistically significant in the MIMIC model specification 8-2-6a. In the two alternative models the signs change from one specification to the other pointing to a less clear pattern and making the empirical results of these two specifications inconclusive. The currency ratio is in neither specification statistically significant. To summarize: In

general, the results are somewhat mixed; only the estimated coefficients of the tax burden and the corporate tax rate are consistent with previous findings of the literature.

Looking at the estimated coefficients of the causes for fiscal illusion we also find, as expected, the tax burden to have a positive, highly statistically significant coefficient. The self-employment rate is not statistically significant, neither is the top income tax rate. The secondary school enrolment rate (education) has the expected negative sign and is statistically significant, confirming our theoretical considerations that a more educated society makes it more difficult for policymakers to effectively delude the true tax burden. Concerning the indicators of fiscal illusion, central government debt as percentage of GDP and the ratio of indirect taxes to direct taxes show unexpected signs in the MIMIC model specification 8-2-6a. The signs are however as expected in specification 8-2-6b but due to the lack of robustness we do not consider these results as conclusive. The Herfindahl index of tax system complexity has the expected, highly statistically significant negative signs in all estimated models.

Finally, we turn to interpret the coefficients indicating the influence from the shadow economy to fiscal illusion and vice versa. While the coefficient describing the influence of the shadow economy on fiscal illusion is positive statistically significant, the coefficient describing the influence of fiscal illusion on the shadow economy is negative statistically significant. When interpreting the negative link between fiscal illusion and the shadow economy one should take into account the fact that an increasing tax burden contributes to more shadow economic activities and – at the same time – encourages the government to implement a tax policy aimed to increase fiscal illusion. Thus, the rationale behind the result of a higher level of fiscal illusion decreasing the shadow economy potentially is that policymakers aim to reduce the incentives for tax evasion

through fiscal illusion mainly in countries with a large shadow economy. One measure policymakers may use, which is empirically supported by our model, is to systematically delude taxpayers.

For the link from the shadow economy to fiscal illusion, we find a positive significant effect; an increase of shadow economic activities increases fiscal illusion, all other things being equal. The rationale for this finding may be that a large shadow economy potentially erodes the quality of institutions and people's appreciation for the state. Policymakers who lose credit may wish to use several strategies to limit the size of the shadow economy. In this sense, policymakers in countries with larger shadow economies have more incentives to adopt policies to delude the true burden of taxation compared to policymakers in countries with smaller shadow economies and a higher degree of tax compliance. In this way, policymakers may avoid a further increase of tax evasion and shadow economic activities. The available empirical evidence confirms our finding that countries with a higher level of fiscal illusion are often the ones with sizable shadow economies (Dell'Anno and Mourao 2012). An alternative interpretation may be that the shadow economy is just another form of fiscal illusion. In the presence of the shadow economy a difference exists between the official (average) tax burden and the real (average) tax burden. Honest taxpayers thus bear the burden paying to much taxes to compensate the loss of tax revenues through tax evasion and shadow economic activities, i.e., the individual tax burden for honest taxpayers is on average higher than what journalists, politicians and official statistics report.

To summarize: the estimated MIMIC model presents empirical evidence supporting the hypothesis that a higher average tax burden encourages a government to adopt tax policies aimed at increasing the level of fiscal illusion. Further we find a negative

correlation between the level of education and fiscal illusion, i.e., $\beta_{82}<0$. In this sense, the results support the argument that a more educated society reduces the effectiveness of fiscal illusion policies and thus the incentives of policymakers to implement measures to distort taxpayers' perceptions. The self-employment rate, the top statutory personal income tax rate, the Business Freedom index, and the OECD dummy are not statistical different form zero. Regarding the indicators, i.e., the most common strategies to reduce citizens' perceptions of their true tax burden, we find the ratio of public debt to GDP to have the expected sign. Unfortunately, the coefficient is not statistical significant. The estimated coefficient for the second indicator, i.e., the ratio of indirect to direct tax revenue, does not support the Mill hypothesis. Its sign is unexpectedly negative.[10] Finally, the Herfindahl index of a tax and revenue system's complexity is, as expected, negative. Governments, who wish the increase the level of fiscal illusion to let taxpayers underestimate the true tax burden, seem to design more complicated and complex tax and revenue systems, all other things being equal.

4. Conclusion

This paper aims to extend the empirical literature on the relationship between fiscal illusion and the shadow economy, making the attempt to simultaneously estimate both latent variables in a MIMIC model using a sample of 104 developed and developing countries. Our empirical results show that a higher statutory tax burden positively contributes to the size and development of the shadow economy and incentivizes the government to increase the level of fiscal illusion. This policy is however less effective

[10] On the contrary, all three indicators of fiscal illusion have the expected signs in the alternative model specification MIMIC 8-2-6b and with exception of public debt also in specification MIMIC 6-2-6.

the higher the educational level of citizens on average is. Most importantly, this paper presents for the first time a simultaneous estimation of the two latent variables shadow economy and fiscal illusion. The estimated coefficients of the relationship between those two indicate that the shadow economy positively impacts fiscal illusion, that is, an increase of the shadow economy also leads to an increase of fiscal illusion. The reason may be a deterioration of the quality of institutions and attitudes towards the state. Policymakers may react to this trend using strategies to delude the true tax burden in order to deplete shadow economic activities. Alternatively, shadow economic activities may be just another form of fiscal illusion. Fiscal illusion though negatively impacts the shadow economy, meaning that higher levels of fiscal illusion decrease the shadow economy: Policymakers obviously aim to reduce the incentives for tax evasion and shadow economic activities through illusion, which is probably a fair strategy mainly in countries where the shadow economy sector is sizable.

In general, our empirical results are promising but have to be interpreted with great caution, as they are not as robust as we would have liked them and should thus be seen as a first step only. However, from a methodological viewpoint, the approach utilized in our paper underlines the complexity of the relationship between the two phenomena fiscal illusion and the shadow economy. In this sense, it highlights the need to apply systematic statistical approaches, such as MIMIC modeling techniques, to investigate the nature of these latent phenomena.

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Appendix A: Data sources

	Name	Source	Mean	Obs.
X_1	Personal Income Tax	Personal Income Tax to GDP; IMF GFS Database, OECD Revenue Statistics, CEPAL	5.16	1566
X_2	Corporate Income Tax	Corporate Income Tax to GDP; IMF GFS Database, OECD Revenue Statistics, CEPAL	2.83	1633
X_3	Unemployment	Unemployment rate; WDI database	8.30	1547
X_4	Business Freedom	Business freedom; Heritage Foundation	68.37	1494
X_5	Tax Revenue	Total Revenues to GDP; IMF GFS Database, OECD Revenue Statistics, CEPAL	29.38	1713
X_6	Self-Employment	Total self-employed workers as a proportion of total employment; WDI	28.33	1370
X_7	Top personal income tax rate	Top statutory PIT rate (%); World Tax Indicators, International Center for Public Policy	35.49	1604
X_8	Education	School enrolment, secondary (% gross); WDI	78.75	1718
X_9	OECD	Dummy variable: 1 if OECD countries and 0 otherwise	0.29	2184
Y_1	Labor force part.	Labor force participation rate; WDI	62.67	2184
Y_2	Growth GDP	GDP per capita growth, annual (%); WDI	1.80	2163
Y_3	Currency ratio	M0 over M1; IMF - International Financial Statistics	2.03	1774
Y_4	Normalized Herfindahl Index of Revenue	$\{[\text{Taxes on goods and services (\% of revenue)}^2 + \text{taxes on income (\% revenue)}^2 + \text{social security contributions (\% revenues)}^2 + \text{other taxes (\% of revenue)}^2] + [100 - (\text{Taxes on goods and services (\% of revenue)} + \text{taxes on income (\% revenue)} + \text{social security contributions (\% revenues)} + \text{other taxes (\% of revenue)})]^2 - 1/5)\}/[10000-(1/5)]$; WDI	0.36	1314
Y_5	Public Debt	Central government debt, total (% of GDP); WDI	58.18	866
Y_6	Indirect Tax / Direct Tax	Ratio of indirect and direct tax revenues (i.e., ratio of taxes on goods and services / taxes on income, profits and capital gains); WDI	1.88	1308

List of countries: Australia, Austria, Bangladesh, Belgium, Belize, Benin, Bhutan, Bolivia, Brazil, Bulgaria, Burkina Faso, Burundi, Canada, Chile, China, P.R., Hong Kong, Colombia, Congo Dem. Rep., Congo Rep., Costa Rica, Côte d'Ivoire, Croatia, Cyprus, Czech Republic, Denmark, Dominican Republic, Ecuador, Egypt, El Salvador, Estonia, Ethiopia, Fiji, Finland, France, Georgia, Germany, Greece, Guatemala, Honduras, Hungary, Iceland, India, Indonesia, Iran, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kazakhstan, Kenya, Korea S., Kuwait, Latvia, Lesotho, Lithuania, Luxembourg, Madagascar, Malaysia, Malta, Mauritius, Mexico, Moldova, Mongolia, Morocco, Namibia, Nepal, Netherlands, New Zealand, Nicaragua, Norway, Oman, Pakistan, Panama, Paraguay, Peru, Philippines, Poland, Portugal, Romania, Russia, Senegal, Sierra Leone, Singapore, Slovak Republic, Slovenia, South Africa, Spain, Sri Lanka, Swaziland, Sweden, Switzerland, Syrian Arab Republic, Thailand, Trinidad and Tobago, Tunisia, Turkey, Uganda, Ukraine, United Kingdom, United States, Uruguay, Venezuela, Zimbabwe.

Appendix B: RAM representation of MIMIC 8-1-6a

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{24} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{25} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{26} \\ \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} & \beta_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta_{12} \\ 0 & 0 & 0 & \beta_{24} & \beta_{25} & \beta_{26} & \beta_{27} & \beta_{28} & 0 & 0 & 0 & 0 & 0 & 0 & \eta_{21} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \\ \zeta_1 \\ \zeta_2 \end{bmatrix}$$

$$P = \begin{bmatrix} \sigma_{11} & & & & & & & & & & & & & & & \\ \sigma_{21} & \sigma_{22} & & & & & & & & & & & & & & \\ 0 & 0 & \sigma_{33} & & & & & & & & & & & & & \\ 0 & 0 & 0 & \sigma_{44} & & & & & & & & & & & & \\ \sigma_{51} & \sigma_{52} & 0 & 0 & \sigma_{55} & & & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & \sigma_{66} & & & & & & & & & & \\ 0 & 0 & 0 & 0 & \sigma_{75} & 0 & \sigma_{77} & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{88} & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta^{\varepsilon}_{11} & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta^{\varepsilon}_{21} & \theta^{\varepsilon}_{22} & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta^{\varepsilon}_{33} & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta^{\varepsilon}_{42} & 0 & \theta^{\varepsilon}_{44} & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta^{\varepsilon}_{55} & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta^{\varepsilon}_{66} & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{21} & 1 \end{bmatrix}$$

Where: σ_{ii} is the variance of the observable cause x_i; $\theta^{\varepsilon}_{ii}$ is the variance of the measurement error ε_i; $\psi_{11} = \psi_{22}$ are the variances of the latent variables *f_1 and f_2* fixed to be equal to 1; all elements off the diagonal are the respective covariances.