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by

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Abstract

Inherited wealth creates a second distinguishing characteristic of individuals, in addition to earning abilities. We incorporate this fact into a model of optimal labor income taxation, with bequests motivated by joy of giving. We find that taxes on bequests or on inheritances allow further redistribution, if in the parent generation initial wealth and earning abilities are positively related. On the other hand, these taxes distort the bequest decision; thus, the overall effect on social welfare is ambiguous. A tax on all expenditures of a generation (a uniform tax on consumption plus bequests) has the same redistributive effect as an inheritance tax but does not distort the bequest decision.

JEL-classification: H21, H24.
Keywords: optimal taxation, inheritance tax, expenditure tax, intergenerational wealth transfer.

I. Introduction

The tax on estates or inheritances has always been a highly controversial issue, particularly in recent years, when it was abolished in some countries like Sweden and Austria.[1] In the political discussion, opponents consider it morally inappropriate to use the moment of death as a cause for imposing a tax, and stress its negative economic consequences, in particular on capital accumulation and on family business. Supporters find these consequences exaggerated and claim that a tax on bequests is desirable for redistributive reasons, contributing to "equality of opportunity".

From the perspective of economic theory the essential issue is to formulate an appropriate model, which allows a discussion of how a shift from labor income taxation to a tax on intergenerational wealth transfers affects social welfare. In this paper we suggest an extended optimal-taxation model in the tradition of Mirrlees (1971). The starting point of our analysis is the following: inherited wealth creates a second distinguishing characteristic of individuals, in addition to earning abilities. The role of taxes on estates or inheritances cannot be well understood without an explicit consideration of this fact.

Typically former contributions discussing bequest taxation in an optimal-taxation framework have focused on the specifics of leaving bequests, as compared to other ways of spending the budget, that is, to the consumption of goods.[2] Such an analysis, referring to a standard result in optimal-taxation theory (Atkinson and Stiglitz 1976), leads to the question of whether preferences are separable between leisure and consumption plus bequests – then an income tax alone suffices, spending need not be taxed at all – or whether leaving bequests represents a complement or a substitute to enjoying leisure. We argue in the present paper that this is the inappropriate question, because the Atkinson-Stiglitz result is derived for a model where individuals only differ in earning abilities. What matters is not that bequests represent a particular use of the budget, but the fact that they generate differences in inherited wealth within a generation.

[1] But this tax still exists in most OECD countries.

[2] See Gale and Slemrod (2001, p.33) and Kaplow (2001), as well as Blumkin and Sadka (2003) in the context of a dynastic model.

We consider an optimal-taxation model which accounts for these differences, in addition to earning abilities, and analyze the welfare effect of proportional taxes on bequests or inheritances, as well as of a proportional tax on all expenditures (i.e. on consumption plus bequests), which can be imposed in addition to an optimal nonlinear income tax. While we have studied a similar model (Brunner and Pech, forthcoming) with bequests motivated by pure altruism, we now assume a joy-of-giving motive for leaving bequests. As is well-known, the altruistic motive leads to the study of dynasties (Barro 1974) where preferences of parents comprise preferences of all descendant generations. A joy-of-giving motive constitutes a weaker link between generations; individuals just enjoy bequeathing some amount, without an explicit reference to their heirs' welfare position. Empirical studies find quite mixed results (for an overview, see Kopczuk and Lupton 2007, Kopczuk 2010) and thus do not allow for a definitive conclusion which of the two bequest motives dominates the actual decisions. Instead it appears likely that a combination of these bequest motives (together with strategic and accidental bequests)[3] is required to explain the observed pattern of bequests. Consequently, the investigation of the effect of taxing bequests, when the latter are motivated by joy of giving, is important.

To some extent, we find that the consequences of a tax on bequests or on inheritances are similar, irrespective of whether an altruistic or a joy-of-giving motive prevails. Both generate a positive welfare effect by allowing increased redistribution (compared to an income tax alone), if initial (inherited) wealth is positively related to earning abilities. Indeed, this effect may be the reason why these taxes are frequently considered as increasing equality of opportunity; it does not occur in a model without differences in initial wealth. On the other hand, however, the bequest motive makes a major difference for the expenditure tax. Generally, taxing the expenditures of an individual of some generation t is equivalent to taxing her initial wealth, if combined with an optimal nonlinear labor income tax. With an altruistic motive, this equivalence extends to the distorting effect on the previous generation $t-1$, whose bequests generate initial wealth of generation t. But with a joy-of-giving motive, this equivalence does not hold any more; the expenditure

[3]For a classification of bequest motives, see e.g. Cremer and Pestieau (2006) or Kopczuk (2010).

tax does not distort the bequest decision of generation $t-1$, because the bequeathing individuals do not take into account a tax imposed on expenditures of generation t. As a consequence, the expenditure tax has an unambiguously welfare-increasing effect, if initial wealth is positively related with earning abilities.

There exist some papers which pay attention to the fact that inheritances create a second distinguishing characteristic, in addition to earning abilities. However, to our knowledge this literature does not provide a unified framework for an analysis of the role of bequest taxation within an optimal tax system. Cremer et al. (2001) resume the discussion of indirect taxes (in addition to an optimal nonlinear income tax), given that individuals differ in endowments as well as abilities. They assume, however, that inheritances are unobservable and concentrate on the structure of indirect tax rates. Similarly, Cremer et al. (2003) and Boadway et al. (2000) study the desirability of a tax on capital income as a surrogate for the taxation of inheritances, which are considered unobservable.

In contrast to these contributions, we study a comprehensive tax system where a nonlinear tax on labor income can be combined with taxes on inherited wealth and on expenditures. Therefore, we take all these variables as being observable (only abilities are unobservable). This is indeed the basis upon which real-world tax systems, including the tax on bequests, operate. In particular, notwithstanding problems of observability, if we want to know whether the inheritance tax should be retained or abolished from a welfare-theoretic point of view, the analysis must be based on the assumption of observable initial wealth.

In the following Section II we introduce the basic model and discuss the consequences of bequest taxation, given that initial wealth is exogenous. In Section III we incorporate the distortion of the bequest decision of the previous generation. Section IV provides some discussion of the results.

II. The model

We start with the simplest model that allows us to analyze the consequences of bequest taxation on social welfare, including redistribution. In some period t there exist two individuals $i = L, H$ with abilities $\omega_{Lt} < \omega_{Ht}$. Individuals live for one period only. By supplying labor l_{it} they earn gross income $z_{it} = l_{it}\omega_{it}$ and net income x_{it}. They are endowed with initial wealth e_{it}, which is used, together with net income, for own consumption c_{it} and for leaving bequests b_{it}. For the moment we take initial wealth as exogenous, it resulted from bequests left by the previous generation, whose behavior will be studied in the next section. Preferences are identical for the individuals and can be described by the strictly concave utility function $u(c_{it}, b_{it}, l_{it})$, which is assumed to be twice differentiable with $\partial u/\partial c_{it} > 0, \partial u/\partial b_{it} > 0, \partial u/\partial l_{it} < 0$. Bequests are assumed to be a normal good.

In period $t+1$ there exists a generation of heirs, which again consists of two individuals. They do not work but live on inheritances e_{jt+1}, which are equal to consumption c_{jt+1}, $j = 1, 2$. Parents determine how bequests b_{it} are assigned to inheritances e_{jt+1}. In contrast to the model with altruistic preferences, where transmission necessarily takes place from a parent to the child within a dynasty, in the present model parents may split their estates and leave some share to each individual of the next generation. As our results hold for any such assignment, we need not specify this decision in detail. We just describe it as a linear rule, $e_{jt+1} = \alpha_{jLt}b_{Lt} + \alpha_{jHt}b_{Ht}$, with $\alpha_{jit} \geq 0$, $\sum_{j=1}^{2} \alpha_{jit} = 1, i = L, H$, and write generally $e_{jt+1}(b_{Lt}, b_{Ht})$ or $c_{jt+1}(b_{Lt}, b_{Ht})$ to indicate the dependency. Utility out of consumption in period $t + 1$ is described by $U(c_{jt+1})$.

The government imposes a nonlinear tax on labor income in period t. We will analyze the role of three tax instruments which the government can use in addition, namely a proportional tax τ_{bt} on bequests b_{it}, a proportional tax τ_{et} on initial wealth e_{it}, and a proportional tax τ_t on total expenditures $c_{it} + b_{it}$.

Indirect utility of an individual i of generation t is defined for given values of gross income z_{it}, net income x_{it}, initial wealth e_{it} and given tax rates τ_{bt}, τ_{et} and τ_t:

$$\begin{aligned} & v_t^i(x_{it}, z_{it}, e_{it}, \tau_{bt}, \tau_{et}, \tau_t) \equiv \\ & \quad \max\{u(c_{it}, b_{it}, z_{it}/\omega_{it}) | (1 + \tau_t)(c_{it} + (1 + \tau_{bt})b_{it}) \leq x_{it} + (1 - \tau_{et})e_{it}\}, \end{aligned} \tag{1}$$

where we assume that only one of τ_{bt} and τ_t exists. The RHS of (1) also gives us the demand functions c_{it} and b_{it} with the same arguments as v_t^i. In this framework, finding the optimal nonlinear income tax is equivalent to determining the optimal gross- and net-income bundles (x_{Lt}, z_{Lt}) and (x_{Ht}, z_{Ht}). We assume that the government follows a utilitarian objective with individual weights f_{it}, f_{jt+1} and requires tax revenues g_t in period t. Its maximization problem reads as

$$\max_{(x_{it}, z_{it}), i=L,H} \sum_{i=L,H} f_{it} v_t^i(\cdot) + (1+\gamma)^{-1} W_{t+1}(b_{Lt}, b_{Ht}), \tag{2}$$

$$\text{s.t. } v_t^H(x_{Ht}, z_{Ht}, e_{Ht}, \tau_{bt}, \tau_{et}, \tau_t) \geq v_t^H(x_{Lt}, z_{Lt}, e_{Ht}, \tau_{bt}, \tau_{et}, \tau_t), \tag{3}$$

$$\sum_{i=L,H} x_{it} \leq \sum_{i=L,H} z_{it} + \tau_{et} \sum_{i=L,H} e_{it} + \tau_{bt} \sum_{i=L,H} b_{it} + \tag{4}$$

$$+ \tau_t \sum_{i=L,H} (c_{it} + b_{it}) - g_t.$$

In the objective we have abbreviated $\sum_{j=1}^{2} f_{jt+1} U(c_{jt+1}(b_{Lt}, b_{Ht}))$ by $W_{t+1}(b_{Lt}, b_{Ht})$, and the coefficient γ denotes the social discount rate of the welfare of the future generation. It can also be interpreted as expressing the degree of double-counting of the bequests in the social objective, because bequests enter utility of generation t and $t+1$ (note that consumption c_{jt+1} is financed by b_{it}).

(4) is the public resource constraint and (3) represents the self-selection constraint, where we assume that in the optimum only the self-selection constraint for the high-able individual is binding, so that the corresponding one for the low-able individual can be neglected. Moreover we assume that the single-crossing condition is fulfilled: $-(\partial v_t^i(\cdot)/\partial z_{it})/(\partial v_t^i(\cdot)/\partial x_{it})$ is larger for type L than for type H, at any admissible (x, z)-bundle.

As is standard in studies of indirect taxes in a Mirrlees-type model, we assume that the tax authority cannot identify individuals through information obtained when collecting the taxes on bequests, initial wealth or expenditures. Otherwise, given a fixed relation between the individuals' abilities and the levels of these variables, it would be possible for the government to impose an optimal differentiated lump-sum tax as a first-best solution.[4]

[4] This assumption is in line with actual behavior of tax authorities. Moreover, one can show that the following results remain essentially unchanged, if a stochastic instead of a fixed relation between abilities and the levels of bequests, initial wealth and expenditures is assumed, so that identification is indeed not

Differentiation of the optimal value function $S_t(\tau_{bt}, \tau_{et}, \tau_t)$ of (2) - (4) gives us the effect of the three indirect taxes (λ and μ denote the Lagrangian multipliers associated with (3) and (4), respectively):

Proposition 1.

a) Let $\tau_t = 0$. The welfare effect of a tax τ_{bt} on bequests left by generation t reads as

$$\frac{\partial S_t}{\partial \tau_{bt}} = \mu \frac{\partial v_t^H}{\partial x_{Lt}}(b_{Ht}[L] - b_{Lt}) + (1+\gamma)^{-1} \sum_{i=L,H} \frac{\partial W_{t+1}}{\partial b_{it}} \frac{\partial b_{it}^{com}}{\partial \tau_{bt}} + \lambda \tau_{bt} \sum_{i=L,H} \frac{\partial b_{it}^{com}}{\partial \tau_{bt}}.$$

b) Let $\tau_{bt} = 0$. The welfare effect of a tax τ_{et} on initial wealth of generation t reads as

$$\frac{\partial S_t}{\partial \tau_{et}} = \mu \frac{\partial v_t^H[L]}{\partial x_{Lt}}(e_{Ht} - e_{Lt}).$$

c) Let $\tau_{bt} = 0$. The welfare effect of a tax τ_t on total expenditure of generation t reads as

$$\frac{\partial S_t}{\partial \tau_t} = \mu \frac{\partial v_t^H[L]}{\partial x_{Lt}}(e_{Ht} - e_{Lt}) \frac{1-\tau_{et}}{1+\tau_t}.$$

Proof. See Appendix. ■

In the above formulas, the upper index "*com*" denotes the compensated demand function, and the symbol $[L]$ indicates mimicking, that is, a situation when the high-able individual opts for the (x_{Lt}, z_{Lt})-bundle designed for the low-able individual. The results of Proposition 1 are analogous to the corresponding ones for a model with an altruistic bequest motive, found in Brunner and Pech, forthcoming. The welfare effects of τ_t and τ_{et} are unambiguously positive, if the high-able individual is endowed with more initial wealth than the low able, as the self-selection constraint is binding (positive multiplier), as is the marginal utility of net income. The interesting point is that both taxes have essentially the same welfare consequences, though the tax τ_{et} on (exogenous) initial wealth is lump-sum while the expenditure tax τ_t is, in principle, distorting, because it falls on total consumption which depends on endogenous labor income. However, it is possible to

possible from these levels (Brunner and Pech 2008).

adjust the optimal tax on labor income in such a way that the distorting effect of τ_t is eliminated. Remarkably, an adjustment of the labor income tax also allows to compensate for all possibly negative effects on bequests of generation t, thus on welfare of generation $t+1$; no such effect occurs in Proposition 1b and 1c.

The positive effect on welfare comes from a relaxation of the self-selection constraint induced by an increase of τ_{et} (or τ_t). The intuition can be explained as follows: assume, as a first-step, that after an increase of τ_{et} by $\Delta\tau_{et}$, each individual i is just compensated through an increase of net labor income x_{it} by $\Delta\tau_{et}e_{it}$. If $e_{Ht} > e_{Lt}$, the high-able individual experiences a larger increase of net labor income than the less able which makes mimicking less attractive and gives slack to the self-selection constraint. As a consequence, in a second step additional redistribution from the high- to the low-able individual becomes possible, which increases social welfare.

The welfare effect is more complex for the tax τ_{bt} on bequests of generation t. If $e_{Ht} > e_{Lt}$ and if preferences are weakly separable between labor and consumption, then we find a positive welfare effect due to increased redistribution, because taxing bequests in fact is a means of taxing initial wealth to some extent. Note that bequests $b_{Ht}[L]$ of the H-type, when mimicking, are larger than bequest b_{Lt} of the L-type, if $e_{Ht} > e_{Lt}$, because bequests are a normal good while weak separability implies that the lower labor time of the mimicker does not change the spending decision (for given net income). Thus, the intuition is similar to the one described above: compensating the individuals by an increase in net income by $\Delta\tau_{bt}b_{Ht}$ and $\Delta\tau_{bt}b_{Lt}$, respectively, gives slack to the self-selection constraint. However, the tax τ_{bt} creates a distortion of the bequest decision, which leads to a - negative - own compensated price effect on bequests and thus affects welfare of generation $t+1$ negatively. Moreover, as the last term in Proposition 1a shows, a deadweight-loss effect for generation t occurs, which is zero at $\tau_{bt} = 0$. Altogether, the welfare effect is ambiguous.

Obviously, if initial wealth is the same for both individuals, then the (positive) redistributive effect disappears for all three taxes. This is the framework of the well-known Atkinson-Stiglitz (1976) result, which tells us that indirect taxes cannot improve social welfare, if they are imposed in addition to an optimal nonlinear labor income tax and individuals differ merely in earning abilities. Our result shows that the role of indirect taxes can be understood when the framework is extended to include the fact that - as a consequence of wealth transmission in previous generations - differences in initial wealth already exist.

III. The previous generation

Next, we go a step back and consider a previous, third generation $t-1$. That is, we take inheritances e_{it} of generation t no longer as exogenous, but consider explicitly the bequest decisions of generation $t-1$. For simplicity we assume again that the previous generation consists of two individuals only, who differ in earning abilities $\omega_{Lt-1} < \omega_{Ht-1}$ and are endowed with initial wealth e_{Lt-1} and e_{Ht-1}. They earn gross income $z_{it-1} = \omega_{it-1} l_{it-1}$ and use net income x_{it-1} together with initial wealth for own consumption c_{it-1} and bequests b_{it-1}. Preferences are again described by the utility function $u(c_{it-1}, b_{it-1}, l_{it-1})$. Bequests of generation $t-1$ constitute the inherited wealth of the next generation t, where we assume as in Section II that each individual of the heir generation t receives some share of the bequests left by each individual of generation $t-1$.

In period $t-1$, the government determines optimal nonlinear income taxes for both periods $t-1$ and t. Moreover, it considers proportional taxes τ_{et} on inheritances and τ_t on expenditures of generation t, which are assumed to be announced at a time such that generation t is able to adapt its behavior. In addition, as an alternative to τ_{et} it considers a tax τ_{bt-1} on bequests of generation $t-1$. We analyze the welfare consequences of these taxes.

How exactly these taxes influence the decision of the generation $t-1$ depends on the bequest motive: in our model, bequests are regarded as some form of consumption; it is the amount left to the heirs, which per se provides utility to the bequeathing individuals. In contrast to the dynastic model, the welfare positions of the descendants are irrelevant for the bequest decision of the parents, and hence also taxes affecting only welfare of the descendant generation are irrelevant.

Still, it has to be discussed how a direct tax τ_{et}, imposed on inherited wealth of generation t, affects the previous generation. Taking the bequest-as-consumption model literally, one might again argue that the anticipation of τ_{et} does not change anything for generation $t-1$, because individuals simply care for what they leave as (gross) bequests to their descendants. On the other hand, however, it seems more plausible to model the bequeathing generation $t-1$ as caring for the amount that actually goes to their heirs, that is, for bequests net of the inheritance tax τ_{et}. Accordingly, we use b_{it-1} as indicating bequests net of an inheritance tax τ_{et}, then gross bequests are $b_{it-1}/(1-\tau_{et})$. This is consistent with our view that individuals also care for net bequests b_{it-1} in case of a bequest tax τ_{bt-1} (here gross bequests are $b_{it-1}(1+\tau_{bt-1})$). Obviously, both taxes impose the same burden if $\tau_{bt-1} = \tau_{et}/(1-\tau_{et})$, then $\tau_{bt-1}b_{it-1} = \tau_{et}(b_{it-1}/(1-\tau_{et}))$. Revenues from the bequest tax τ_{bt-1} (the inheritance tax τ_{et}) are assumed to increase the public budget of generation $t-1$ (generation t, respectively).

Following these considerations, we define indirect utility of generation $t-1$ analogous to (1) as:

$$
\begin{aligned}
&v^i_{t-1}(x_{it-1}, z_{it-1}, e_{it-1}, \tau_{bt-1}, \tau_{et}) \equiv \qquad (5)\\
&\max\{u(c_{it-1}, b_{it-1}, \frac{z_{it-1}}{\omega_{it-1}})|\ c_{it-1} + (1+\tau_{bt-1})\frac{b_{it-1}}{1-\tau_{et}} \leq x_{it-1} + e_{it-1}\},
\end{aligned}
$$

where we assume throughout that only one of τ_{bt-1} and τ_{et} actually exists. (5) also determines demand functions for consumption $c_{it-1}(\cdot)$ and net bequests $b_{it-1}(\cdot)$. By some way of sharing, the latter result in net inheritances $\varepsilon_{it}(b_{Lt-1}, b_{Ht-1})$ of the next generation $t, i = L, H$, such that $\varepsilon_{Lt}+\varepsilon_{Ht} = b_{Lt-1}+b_{Ht-1}$. Indirect utility of individual i of generation t is now defined similar to (1) as

$$
v^i_t(x_{it}, z_{it}, \varepsilon_{it}, \tau_t) \equiv \max\{u(c_{it}, b_{it}, \frac{z_{it}}{\omega_{it}})|(1+\tau_t)(c_{it}+b_{it}) \leq x_{it} + \varepsilon_{it}\}, \qquad (6)
$$

and revenues from the inheritance tax are $\tau_{et}(\varepsilon_{Lt} + \varepsilon_{Ht})/(1-\tau_{et})$.

The intertemporal maximization problem of the government reads as (with weights f_{it-1} for individuals of generation $t-1$, public resource requirements g_{t-1}, and W_{t+1}

defined as in Section II)

$$\max_{\substack{(x_{it-1},z_{it-1}),\\(x_{it},z_{it}),i=L,H}} \sum_{i=L,H} f_{it-1}v^i_{t-1}(\cdot) + (1+\gamma)^{-1}\sum_{i=L,H} f_{it}v^i_t(\cdot) + (1+\gamma)^{-2}W_{t+1}(b_{Lt}, b_{Ht}), \tag{7}$$

$$\text{s.t. } v^H_{t-1}(x_{Ht-1}, z_{Ht-1}, \cdot) \geq v^H_{t-1}(x_{Lt-1}, z_{Lt-1}, \cdot), \tag{8}$$

$$v^H_t(x_{Ht}, z_{Ht}, \cdot) \geq v^H_t(x_{Lt}, z_{Lt}, \cdot), \tag{9}$$

$$\sum_{i=L,H} x_{it-1} \leq \sum_{i=L,H} z_{it-1} + \tau_{bt-1}\sum_{i=L,H} b_{it-1} - g_{t-1}, \tag{10}$$

$$\sum_{i=L,H} x_{it} \leq \sum_{i=L,H} z_{it} + \tau_{et}\sum_{i=L,H} \frac{\varepsilon_{it}}{1-\tau_{et}} + \tau_t \sum_{i=L,H}(c_{it} + b_{it}) - g_t. \tag{11}$$

We now have two self-selection constraints (whose Lagrangian multipliers are denoted by μ_{t-1}, μ_t) and two resource constraints (multipliers λ_{t-1}, λ_t). Note that in this maximization problem the government is assumed to have no instrument for transferring resources across generations (see the remark after Proposition 3).

We start with an analysis of τ_{bt-1} (ignore τ_{et} and let τ_t be fixed) and consider a subproblem of (7) - (11) by defining for given b_{Lt-1} and b_{Ht-1}

$$W_t(b_{Lt-1}, b_{Ht-1}, \tau_t) \equiv \tag{12}$$

$$\max_{x_{it},z_{it},i=L,H} \sum_{i=L,H} v^i_t(x_{it}, z_{it}, \varepsilon_{it}(b_{Lt-1}, b_{Ht-1}), \tau_t) + (1+\gamma)^{-1}W_{t+1}(b_{Lt}, b_{Ht})$$

$$\text{s.t. (9) and } \sum_{i=L,H} x_{it} \leq \sum_{i=L,H} z_{it} + \tau_t \sum_{i=L,H}(c_{it} + b_{it}) - g_t.$$

Then we can rewrite the problem (7) - (11) as

$$\max_{x_{it-1},z_{it-1},i=L,H} \sum_{i=L,H} v^i_{t-1}(\cdot) + (1+\gamma)^{-1}W_t(b_{Lt-1}, b_{Ht-1}, \tau_t), \tag{13}$$

$$\text{s.t.} \quad \text{(8) and (10).}$$

In this formulation, the problem is completely the same as the earlier one (2) - (4) for period t. Hence we get (let S_{t-1} denote the optimal value function of (7) - (11)):

Proposition 2.

The welfare effect of a tax τ_{bt-1} on bequests left by generation $t-1$ reads as

$$\frac{\partial S_{t-1}}{\partial \tau_{bt-1}} = \mu_{t-1}\frac{\partial v_{t-1}^{H}}{\partial x_{Lt-1}}(b_{Ht-1}[L]-b_{Lt-1}) + \\ +(1+\gamma)^{-1}\sum_{i=L,H}\frac{\partial W_t}{\partial b_{it-1}}\frac{\partial b_{it-1}^{com}}{\partial \tau_{bt-1}} + \lambda_{t-1}\tau_{bt-1}\sum_{i=L,H}\frac{\partial b_{it-1}^{com}}{\partial \tau_{bt-1}}.$$

Proof. Using W_t (defined in (12)) instead of W_{t+1}, the proof is completely analogous to the proof of Proposition 1a with $t-1$ instead of t. ■

As expected, the welfare effect of a tax τ_{bt-1} on bequests of generation $t-1$ is the same as already found in Proposition 1, now occurring one period earlier. There is a positive effect, given that $e_{Ht-1} > e_{Lt-1}$ (and weak separability of the preferences), because the tax allows increased redistribution through a relaxation of the self-selection constraint, and a negative compensated effect on bequests available for the future generation, as well as negative deadweight-loss effect (as soon as $\tau_{bt-1} > 0$).

It should also be noted that, as a consequence of the specific bequest motive, a tax τ_{bt} on bequests left by generation t does not affect the previous generation $t-1$, and the welfare effect found in Proposition 1 remains valid also in the three-generations model.

As an alternative to the bequest tax τ_{bt-1} we now analyze the welfare effect of a tax τ_{et} on inheritances of the following generation, and of an expenditure tax τ_t; in Section II these two taxes were shown to be completely equivalent in a model with exogenous inheritances of generation t. We again consider a subproblem of (7) - (11) by defining for given b_{Lt-1} and b_{Ht-1}

$$\tilde{W}_t(b_{Lt-1}, b_{Ht-1}, T_{et}, \tau_t) \equiv \\ \max_{x_{it}, z_{it}, i=L,H} \sum_{i=L,H} v_t^i(x_{it}, z_{it}, \varepsilon_{it}, \tau_t) + (1+\gamma)^{-1}W_{t+1}(b_{Lt}, b_{Ht}), \tag{14}$$

$$\text{s.t.} \quad v_t^H(x_{Ht}, z_{Ht}, \varepsilon_{Ht}, \tau_t) \geq v_t^H(x_{Lt}, z_{Lt}, \varepsilon_{Ht}, \tau_t), \tag{15}$$

$$\sum_{i=L,H} x_{it} \leq \sum_{i=L,H} z_{it} + T_{et} + \tau_t \sum_{i=L,H}(c_{it} + b_{it}) - g_t, \tag{16}$$

where $T_{et} \equiv \tau_{et}\sum_{i=L,H}\varepsilon_{it}/(1-\tau_{et})$ are the tax revenues. Using $\tilde{W}_t$ instead of W_t we arrive at the same formulation (13), (8), (10) (now without revenues from τ_{bt-1}) of the government's problem. The difference between W_t and $\tilde{W}_t$ is that while the bequest tax τ_{bt-1} affects generation t only via its influence on bequests b_{it-1} (but tax revenues go to generation $t-1$), revenues of the inheritance tax τ_{et} go to generation t (T_{et} is an argument of $\tilde{W}_t$). Otherwise, the similarity to the earlier problem (2) - (4) remains.

Proposition 3.

a) The welfare effect of a tax τ_{et} on inheritances of generation t reads as

$$\begin{aligned}\frac{\partial S_{t-1}}{\partial \tau_{et}} &= \{\mu_{t-1}\frac{\partial v_{t-1}^{H}}{\partial x_{Lt-1}}(b_{Ht-1}[L]-b_{Lt-1})+\\ &+(1+\gamma)^{-1}[\sum_{i=L,H}\frac{\partial \tilde{W}}{\partial b_{it-1}}\frac{\partial b_{it-1}^{com}}{\partial \tau_{bt-1}}+\frac{\tau_{et}}{1-\tau_{et}}\frac{\partial \tilde{W}}{\partial T_{et}}\sum_{i=L,H}\frac{\partial b_{it-1}^{com}}{\partial \tau_{bt-1}}]-\\ &-\lambda_{t-1}\sum_{i=L,H}b_{it-1}+(1+\gamma)^{-1}\frac{\partial \tilde{W}}{\partial T_{et}}\sum_{i=L,H}b_{it-1}\}\frac{1}{(1-\tau_{et})^2}.\end{aligned}$$

b) The welfare effect of a tax τ_t on expenditures of generation t reads as

$$\frac{\partial S_{t-1}}{\partial \tau_t}=\frac{\partial \tilde{W}_t}{\partial \tau_t}=\mu_t\frac{\partial v_t^H[L]}{\partial x_{Lt}}(e_{Ht}-e_{Lt})\frac{1-\tau_{et}}{1+\tau_t}.$$

Proof. See Appendix. ■

Noting the equivalence of the tax burden of τ_{et} and of an ad-valorem tax on bequests with rate $\tau_{bt-1}=\tau_{et}/(1-\tau_{et})$, the derivative with respect to τ_{et} was replaced by the derivative with respect to τ_{bt-1}. Then it can be seen by comparing Proposition 2 and the first two lines in Proposition 3a that indeed the welfare effect of the tax τ_{et} on inheritances of generation t is similar to the effect of τ_{bt-1}. The difference is that the deadweight-loss, occurring with $\tau_{et}>0$ affects the resources of the descendant generation t negatively, as revenues go into this generation's budget. The two expressions in the third line of Proposition 3a represent the negative effect of τ_{et} on resources of generation $t-1$ and the positive effect on resources of generation t. One can show that if the government has an instrument for shifting resources over generations, we have $(1+\gamma)^{-1}\partial\tilde{W}/\partial T_{et}=\lambda_{t-1}$ and these two effects outweigh each other.[5]

As already discussed above, taxes that only affect welfare of generation t do not influence the behavior of the previous generation. Hence, the tax τ_t on expenditures of generation t has no distorting effect on the bequest decision of generation $t-1$, and it follows that its welfare effect is unambiguously positive, the same as found in Proposition 1. Let us also mention that, given the analogous structure of the maximization problems for generation $t-1$ and t, respectively, a tax τ_{et-1} on inheritances of generation $t-1$ or a tax τ_{t-1} on all expenditures of generation $t-1$ have the analogous welfare effects as described in Proposition 1 for generation t.

We close this section by a short discussion on the optimal tax rates. Assuming that the introduction of a tax τ_{bt-1} or a tax τ_{et} has a positive welfare effect (the formulas in Propositions 2 and 3a are positive when evaluated at $\tau_{bt-1} = 0$ and $\tau_{et} = 0$, respectivley), one can find a characterization of the optimal tax rates from the condition that the deadweight loss offsets the welfare effect.[6] For the expenditure tax τ_t no distortion of the bequest decision occurs, hence there is no balancing effect in our model. In this case, to determine an optimal rate would require an extension of the model, e.g. by allowing tax avoidance (such a model was studied in Brunner et al. 2010).

IV. Discussion

In the present paper we have studied whether a marginal shift from labor taxation to taxes on the intergenerational transmission of wealth increases social welfare, given that leaving bequests is motivated by joy of giving. For a tax on bequests, imposed on the

[5] Obviously, the intuition provided for τ_{bt} (see the paragraphs following Proposition 1) applies in the same way for the redistributive effect of τ_{bt-1} within generation $t-1$. To see the redistributive effect of τ_{et}, also within generation $t-1$ (though its revenues go to generation t), note that a marginal increase $\Delta\tau_{et}$ raises tax revenues $\Delta\tau_{et}(b_{Lt-1} + b_{Ht-1})/(1-\tau_{et})$. Imagine that beforehand the government shifts this marginal amount $\Delta\tau_{et}(b_{Lt-1}+b_{Ht-1})/(1-\tau_{et})$ in a lump-sum way from generation $t-1$ to generation t, by reducing resources available for $x_{Lt-1}+x_{Ht-1}$ and increasing resources available for $x_{Lt}+x_{Ht}$. The corresponding welfare effect is visible in the third line of a), it is negative for generation $t-1$ (multiplier λ_{t-1}) and positive for generation t (marginal social welfare weight $(1+\gamma)^{-1}\partial\tilde{W}/\partial T_{et}$). Then we can think of tax revenues from $\Delta\tau_{et}$ as remaining within generation $t-1$, and exactly the intuition described earlier for τ_{bt} explains the redistributive effect of τ_{et}.

[6] Compare the approach to derive the formulas characterizing the optimal tax rates in the model with an altruistic bequest motive in Brunner and Pech, forthcoming.

parent generation, and a tax on inheritances, imposed on the heirs (but anticipated by the parents), we have found that these taxes indeed allow further redistribution, on top of what can be attained by a labor income tax alone, if in the parent generation initial wealth of the more-able individual is larger than that of the less-able individual. On the other hand, these taxes distort the bequest decision; as a consequence, the overall effect on social welfare is ambiguous. In contrast to the result found in the model with altruistic preferences, these two taxes are not completely equivalent, because it matters which generation receives the tax revenues (unless the government has an instrument for shifting resources over time).

A major difference between the two models arises for the expenditure tax. With an altruistic motive, this tax is completely equivalent to an inheritance tax, both have the two effects just described. With a joy-of-giving motive, however, it turns out that the distorting effect does not occur with an expenditure tax. To see the reason note that an inheritance tax on generation t clearly has a distorting effect on the bequest decisions of the previous generation $t-1$. Now, as we have seen, an expenditure tax on generation t also taxes - indirectly - inheritances of generation t; it increases social welfare of this generation by allowing further redistribution, given again that inherited wealth of the high-able individual is larger than that of the low-able individual. On the other hand, with a joy-of-giving motive individuals of generation $t-1$ only care about the amount left to their descendants. They do not, by definition, care about taxes imposed on the expenditures of generation t, even if these expenditures are (partly) financed out of the bequests they leave. Therefore no distorting effect on their bequest decision occurs.

The essential point here is foresight of the individuals. In the altruistic model it is assumed that parents have perfect foresight of all factors influencing the descendants' utility positions, which directly enter the parents' preferences. Thus also taxes imposed on expenditures of the heirs matter for the parents' decision. No such foresight is assumed with the joy-of-giving motive; only factors (like a bequest or inheritance tax) which directly affect the bequeathed amount distort the parents' decision.

Ultimately, which model is closer to reality is an empirical question, but it seems to be a quite difficult task to test the extent of foresight over generations. Empirical studies trying to discriminate between the altruistic and the joy-of-giving motive analyze whether parents differentiate bequests according to the recipient child's welfare position, which clearly is a different question than foresight of future taxes.

Our starting point was that in view of the Atkinson-Stiglitz result the role of indirect taxes can only be understood in a model where - as a consequence of wealth transmission over prior generations - differences in initial wealth exist, in addition to differences in abilities. Then indirect taxes allow more redistribution than a labor income tax alone, provided that initial wealth of the more able individual is larger than that of the less able individual. Indeed, if differences in initial wealth are neglected, then double-counting of the benefits of bequests (which increase parent utility and have a positive external effect on the heirs) leads to the reverse result that a subsidy of bequests is unambiguously desirable (see Kaplow 2001, Blumkin and Sadka 2003, Farhi and Werning 2010).[7] As already mentioned, in our model the extent of double-counting is indicated by the social discount rate γ. The larger γ, the lower is the weight of the latter in the social welfare function, and, as Propositions 1a, 2 and 3a show, the more probable is it that the positive welfare effect of the introduction of a bequest or inheritance tax dominates the negative effect on the next generation.

Finally, it should be noted that our analysis remains essentially unchanged, if an arbitrary number of generations is considered. On the one hand it is obvious that with a joy-of-giving motive, taxes imposed on inheritances of some generation t never influence bequest decisions of generations earlier than $t-1$. On the other hand, it is always possible to follow the procedure applied in Section III, that is, to define a function W that depends on bequests of some generation and captures their welfare consequences for all future generations. Thus, the model underlying Propositions 2 and 3 is indeed so general as to allow studying the welfare effects of inheritance taxation for arbitrarily many future generations.

[7] From the perspective of practical economic policy this is a very surprising conclusion. Our appoach puts it into perspective by revealing the redistributive potential of bequest taxation, and provides a theoretical underpinning for the frequently expressed view that bequest taxation enhances "equality of opportunity".

V. Appendix

Proof of Proposition 1

From the Lagrangian to the maximization problem (2) - (4) we derive the first-order conditions with respect to x_{Lt} and x_{Ht},

$$f_{Lt}\frac{\partial v_t^L}{\partial x_{Lt}} + (1+\gamma)^{-1}\frac{\partial W_{t+1}}{\partial b_{Lt}}\frac{\partial b_{Lt}}{\partial x_{Lt}} - \mu\frac{\partial v_t^H[L]}{\partial x_{Lt}} - \lambda + \lambda\tau_{bt}\frac{\partial b_{Lt}}{\partial x_{Lt}} + \tag{A1}$$
$$+ \lambda\tau_t(\frac{\partial c_{Lt}}{\partial x_{Lt}} + \frac{\partial b_{Lt}}{\partial x_{Lt}}) = 0$$
$$f_{Ht}\frac{\partial v_t^H}{\partial x_{Ht}} + (1+\gamma)^{-1}\frac{\partial W_{t+1}}{\partial b_{Ht}}\frac{\partial b_{Ht}}{\partial x_{Ht}} + \mu\frac{\partial v_t^H}{\partial x_{Ht}} - \lambda + \lambda\tau_{bt}\frac{\partial b_{Ht}}{\partial x_{Ht}} + \tag{A2}$$
$$+ \lambda\tau_t(\frac{\partial c_{Ht}}{\partial x_{Ht}} + \frac{\partial b_{Ht}}{\partial x_{Ht}}) = 0$$

a) Let $\tau_t = 0$. The derivative of the optimal value function S_t with respect τ_{bt} is found by differentiating the Lagrangian:

$$\frac{\partial S_t}{\partial \tau_{bt}} = \sum_{i=L,H}[f_{it}\frac{\partial v_t^i}{\partial \tau_{bt}} + (1+\gamma)^{-1}\frac{\partial W_{t+1}}{\partial b_{it}}\frac{\partial b_{it}}{\partial \tau_{bt}}] + \mu(\frac{\partial v_t^H}{\partial \tau_{bt}} - \frac{\partial v_t^H[L]}{\partial \tau_{bt}}) + \tag{A3}$$
$$+ \lambda\sum_{i=L,H}(b_{it} + \tau_{bt}\frac{\partial b_{it}}{\partial \tau_{bt}}).$$

We use $\partial v_t^i/\partial\tau_{bt} = -b_{it}\partial v_t^i/\partial x_{it}$, $\partial v_t^H[L]/\partial\tau_{bt} = -b_{Ht}[L]\partial v_t^H[L]/\partial x_{Lt}$ and the Slutsky relation $\partial b_{it}/\partial\tau_{bt} = \partial b_{it}^{com}/\partial\tau_{bt} - b_{it}\partial b_{it}/\partial x_{it}$ to transform (A3) to

$$\frac{\partial S_t}{\partial \tau_{bt}} = \sum_{i=L,H}[-f_{it}b_{it}\frac{\partial v_t^i}{\partial x_{it}} + (1+\gamma)^{-1}\frac{\partial W_{t+1}}{\partial b_{it}}(\frac{\partial b_{it}^{com}}{\partial \tau_{bt}} - b_{it}\frac{\partial b_{it}}{\partial x_{it}})] + \tag{A4}$$
$$+ \mu(-b_{Ht}\frac{\partial v_t^H}{\partial x_{Ht}} + b_{Ht}[L]\frac{\partial v_t^H[L]}{\partial x_{Lt}}) + \lambda\sum_{i=L,H}[b_{it} + \tau_{bt}(\frac{\partial b_{it}^{com}}{\partial \tau_{bt}} - b_{it}\frac{\partial b_{it}}{\partial x_{it}})].$$

Multiplying (A1) by b_{Lt} and (A2) by b_{Ht} and adding both to (A4) gives us the formula in Proposition 1a.

b) We set $\tau_{bt} = 0$. Using $\partial v_t^i/\partial\tau_{et} = -e_{it}\partial v_t^i/\partial x_{it}$, $\partial v_t^H[L]/\partial\tau_{et} = -e_{Ht}\partial v_t^H[L]/\partial x_{Lt}$, $\partial c_{it}/\partial\tau_{et} = -e_{it}\partial c_{it}/\partial x_{it}$ and $\partial b_{it}/\partial\tau_{et} = -e_{it}\partial b_{it}/\partial x_{it}$, the welfare effect of τ_{et} can be written as

$$\frac{\partial S_t}{\partial \tau_{et}} = \sum_{i=L,H}[-f_{it}e_{it}\frac{\partial v_t^i}{\partial x_{it}} - (1+\gamma)^{-1}e_{it}\frac{\partial W_{t+1}}{\partial b_{it}}\frac{\partial b_{it}}{\partial x_{it}}] - \mu e_{Ht}(\frac{\partial v_t^H}{\partial x_{Ht}} - \frac{\partial v_t^H[L]}{\partial x_{Lt}}) + \qquad \text{(A5)}$$
$$+\lambda \sum_{i=L,H}[e_{it} - \tau_t e_{it}(\frac{\partial c_{it}}{\partial x_{it}} + \frac{\partial b_{it}}{\partial x_{it}})].$$

Multiplying (A1) and (A2) by e_{Lt} and e_{Ht}, respectively, and substituting into (A5) gives us the formula of Proposition 1b.

c) Let $\tau_{bt} = 0$. Differentiating the Lagrangian of problem (2) - (4) with respect to τ_t gives

$$\frac{\partial S_t}{\partial \tau_t} = \sum_{i=L,H}[f_{it}\frac{\partial v_t^i}{\partial \tau_t} + (1+\gamma)^{-1}\frac{\partial W_{t+1}}{\partial b_{it}}\frac{\partial b_{it}}{\partial \tau_t}] + \mu(\frac{\partial v_t^H}{\partial \tau_t} - \frac{\partial v_t^H[L]}{\partial \tau_t}) + \qquad \text{(A6)}$$
$$+ \sum_{i=L,H}[\lambda(c_{it}+b_{it}) + \lambda\tau_t(\frac{\partial c_{it}}{\partial \tau_t} + \frac{\partial b_{it}}{\partial \tau_t})].$$

The individual budget equation can be written as $c_{it} + b_{it} = B_{it}$, where $B_{it} \equiv (x_{it} + (1-\tau_{et})e_{it})/(1+\tau_t)$. Thus, $\partial c_{it}/\partial \tau_t = (\partial c_{it}/\partial B_{it})(\partial B_{it}/\partial \tau_t) = -(c_{it}+b_{it})\partial c_{it}/\partial x_{it}$ (use $\partial B_{it}/\partial \tau_t = -(x_{it} + (1-\tau_{et})e_{it})/(1+\tau_t)^2 = -(c_{it}+b_{it})/(1+\tau_t)$ and $\partial c_{it}/\partial x_{it} = (\partial c_{it}/\partial B_{it})/(1+\tau_t)$); equivalently $\partial b_{it}/\partial \tau_t = -(c_{it}+b_{it})\partial b_{it}/\partial x_{it}$. Using these relations, together with $\partial v_t^i/\partial \tau_t = -(c_{it}+b_{it})\partial v_t^i/\partial x_{it}$ and $\partial v_t^H[L]/\partial \tau_t = -(c_{Ht}[L] + b_{Ht}[L])\partial v_t^H[L]/\partial x_{Lt}$, in (A6) yields

$$\frac{\partial S_t}{\partial \tau_t} = \sum_{i=L,H}[-f_{it}(c_{it}+b_{it})\frac{\partial v_t^i}{\partial x_{it}} - (1+\gamma)^{-1}(c_{it}+b_{it})\frac{\partial W_{t+1}}{\partial b_{it}}\frac{\partial b_{it}}{\partial x_{it}}] -$$
$$- \mu(c_{Ht}+b_{Ht})\frac{\partial v_t^H}{\partial x_{Ht}} + \mu(c_{Ht}[L] + b_{Ht}[L])\frac{\partial v_t^H[L]}{\partial x_{Lt}} + \qquad \text{(A7)}$$
$$+ \sum_{i=L,H}[\lambda(c_{it}+b_{it}) - \lambda\tau_t(c_{it}+b_{it})(\frac{\partial c_{it}}{\partial x_{it}} + \frac{\partial b_{it}}{\partial x_{it}})].$$

Multiplying (A1) and (A2) by $(c_{Lt}+b_{Lt})$ and $(c_{Ht}+b_{Ht})$, respectively, and substituting into (A7) gives us

$$\frac{\partial S_t}{\partial \tau_t} = \mu\frac{\partial v_t^H[L]}{\partial x_{Lt}}(c_{Ht}[L] + b_{Ht}[L] - c_{Lt} - b_{Lt}). \qquad \text{(A8)}$$

Inserting the (transformed) budget equations of individual H, when mimicking, and of individual L, i. e., $c_{Ht}[L] + b_{Ht}[L] = (x_{Lt} + (1-\tau_{et})e_{Ht})/(1+\tau_t)$ and $c_{Lt} + b_{Lt} = (x_{Lt} + (1-\tau_{et})e_{Lt})/(1+\tau_t)$ into (A8), we obtain the formula of Proposition 1c.

Proof of Proposition 3

a) We use $\tilde{W}_t$ instead of W_t in (13) - where $\tilde{W}_t$ is defined as the optimum of (14), s.t. (15) and (16) - and derive the first-order conditions of the maximization problem (13), (8) and (10) with respect to x_{Lt-1}, x_{Ht-1} (note that T_{et} can be written as $T_{et} = \tau_{et}\sum_{i=L,H} b_{it-1}/(1-\tau_{et})$):

$$f_{Lt-1}\frac{\partial v^L_{t-1}}{\partial x_{Lt-1}} + (1+\gamma)^{-1}(\frac{\partial \tilde{W}_t}{\partial \beta_{Lt-1}}\frac{\partial b_{Lt-1}}{\partial x_{Lt-1}} + \frac{\partial \tilde{W}_t}{\partial T_{et}}\frac{\tau_{et}}{1-\tau_{et}}\frac{\partial b_{Lt-1}}{\partial x_{Lt-1}}) - \\ - \mu_{t-1}\frac{\partial v^H_{t-1}[L]}{\partial x_{Lt-1}} - \lambda_{t-1} = 0, \tag{A9}$$

$$f_{Ht-1}\frac{\partial v^H_{t-1}}{\partial x_{Ht-1}} + (1+\gamma)^{-1}(\frac{\partial \tilde{W}_t}{\partial \beta_{Ht-1}}\frac{\partial b_{Ht-1}}{\partial x_{Ht-1}} + \frac{\partial \tilde{W}}{\partial T_{et}}\frac{\tau_{et}}{1-\tau_{et}}\frac{\partial b_{Ht-1}}{\partial x_{Ht-1}}) + \\ + \mu_{t-1}\frac{\partial v^H_{t-1}}{\partial x_{Ht-1}} - \lambda_{t-1} = 0. \tag{A10}$$

Differentiation of the Lagrangian with respect to τ_{et} gives

$$\frac{\partial S_{t-1}}{\partial \tau_{et}} = \sum_{i=L,H} f_{it-1}\frac{\partial v^i_{t-1}}{\partial \tau_{et}} + (1+\gamma)^{-1}[\sum_{i=L,H}\frac{\partial \tilde{W}}{\partial b_{it-1}}\frac{\partial b_{it-1}}{\partial \tau_{et}} + \frac{\partial \tilde{W}}{\partial T_{et}}(\frac{1}{(1-\tau_{et})^2} \\ \sum_{i=L,H} b_{it-1} + \frac{\tau_{et}}{1-\tau_{et}}\sum_{i=L,H}\frac{\partial b_{it-1}}{\partial \tau_{et}})] + \mu_{t-1}\frac{\partial v^H_{t-1}}{\partial \tau_{et}} - \mu_{t-1}\frac{\partial v^H_{t-1}[L]}{\partial \tau_{et}}). \tag{A11}$$

We find from (5) that $\partial v^i_{t-1}/\partial \tau_{et} = -(b_{it-1}/(1-\tau_{et})^2)\partial v^i_{t-1}/\partial x_{it-1}$, $\partial v^H_{t-1}[L]/\partial \tau_{et} = (b_{Ht-1}[L]/(1-\tau_{et})^2)\partial v^H_{t-1}[L]/\partial x_{Lt-1}$. Moreover, we make use of the equivalence $\tau_{bt-1} = \tau_{et}/(1-\tau_{et})$ to derive $\partial b_{it-1}/\partial \tau_{et} = 1/(1-\tau_{et})^2(\partial b_{it-1}/\partial \tau_{bt-1})$, where $\partial b_{it-1}/\partial \tau_{bt-1} = \partial b^{com}_{it-1}/\partial \tau_{bt-1} - \beta_{it-1}\partial b_{it-1}/\partial x_{it-1}$ (following from the Slutsky decomposition). Using these relations to transform (A11), adding (A9) multiplied by $b_{Lt-1}/(1-\tau_{et})^2$ and (A10) multiplied by $b_{Ht-1}/(1-\tau_{et})^2$, we obtain the formula in Proposition 3a.

b) All $t-1$ variables are independent of τ_t, thus the problem (7) - (11) comes down to problem (14) - (16) for fixed $\varepsilon_{it} = (1-\tau_{et})e_{it}$. Then the proof of Proposition 1c applies.

References

Atkinson, A. B. and Stiglitz, J. E. (1976), The Design of Tax Structure: Direct versus Indirect Taxation, *Journal of Public Economics 6*, 55-75.

Barro, R. J. (1974), Are Government Bonds Net Wealth?, *Journal of Political Economy 82 (6)*, 1095-1117.

Blumkin, T. and Sadka, E. (2003), Estate Taxation with Intended and Accidental Bequests, *Journal of Public Economics 88*, 1-21.

Boadway, R., Marchand, M. and Pestieau P. (2000), Redistribution with Unobservable Bequests: A Case for Taxing Capital Income, *Scandinavian Journal of Economics 102,* 253-267.

Brunner, J. K. and Pech, S. (2008), Optimum Taxation of Inheritances, *Working paper 0806*, Department of Economics, University of Linz.

Brunner, J. K. and Pech, S. (2011), Optimal Taxation of Bequests in a Model with Initial Wealth, *Scandinavian Journal of Economics* (forthcoming).

Brunner, J. K., Eckerstorfer, P. and Pech, S. (2010), Optimal Taxes on Wealth and Consumption in the Presence of Tax Evasion, *Working paper 1004*, Department of Economics, University of Linz.

Cremer, H. and Pestieau, P. (2006), Wealth transfer taxation: a survey of the theoretical literature, in: S. C. Kolm and J. Mercier Ythier, Eds., *Handbook on Altruism, Giving and Reciprocity*, Vol. 2, North Holland, Amsterdam, 1108-1134.

Cremer, H., Pestieau, P. and Rochet, J.-Ch. (2001), Direct versus Indirect Taxation: The Design of the Tax Structure Revisited, *International Economic Review 42,* 781-799.

Cremer, H., Pestieau, P. and Rochet, J.-Ch. (2003), Capital Income Taxation When Inherited Wealth is Not Observable, *Journal of Public Economics 87,* 2475-2490.

Farhi, E. and Werning, I. (2010), Progressive Estate Taxation, *Quarterly Journal of Economics 125*, 635-673.

Gale, W. G. and Slemrod, J. (2001), Overview, in: Gale, W. G., J. R. Hines, Jr. Slemrod, J. (Eds.), *Rethinking Estate and Gift Taxation*, Washington D. C.: Brookings Institution Press, 1-64.

Kaplow, L. (2001), A Framework for Assessing Estate and Gift Taxation, in: Gale, W. G., J. R. Hines, Jr. Slemrod, J. (Eds.), *Rethinking Estate and Gift Taxation*, Washington D. C.: Brookings Institution Press, 164-204.

Kopczuk, W. and Lupton, J. P. (2007), To Leave or Not to Leave: The Distribution of Bequest Motives, *Review of Economic Studies 74*, 207-235.

Kopczuk, W. (2010), Economics of Estate Taxation: A Brief Review of Theory and Evidence, *NBER Working Paper Series 15741*, National Bureau of Economic Research, Cambridge.

Mirrlees, J. (1971), An Exploration in the Theory of Optimum Income Taxation, *Review of Economic Studies 38*, 175-208.