Noisy Information, Interest Rate Shocks and the Great Moderation

by

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Abstract

In this paper we quantitatively evaluate the hypothesis that the Great Moderation is partly the result of a less activist monetary policy. We simulate a New Keynesian model where the central bank can only observe a noisy estimate of the output gap and find that the less pronounced reaction of the Federal Reserve to output gap fluctuations since 1979 can account for half of the reduction in the standard deviation of GDP associated with the Great Moderation. Our simulations are consistent with the empirically documented smaller magnitude and impact of interest rate shocks since the early 1980s.

Keywords: Great Moderation, New Keynesian Model, Noisy Data

JEL codes: E32, E52, E58

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1 Introduction

Since the mid-1980s, the magnitude of fluctuations in economic activity has substantially declined in the U.S. McConnell and Perez-Quiros (2000) and Blanchard and Simon (2001) were among the first to document this so-called Great Moderation (see also Stock and Watson, 2002; Kim et al., 2004). Although there is little doubt that the business cycle has indeed become smoother over time, the sources of the Great Moderation are less clear. Various explanations have been proposed in the literature, including a decline in the volatility of shocks (e.g. Ahmed et al., 2004; Stock and Watson, 2005) and changes in the structure of the economy (e.g. Blanchard and Simon, 2001; Dynan et al., 2006; Davis and Kahn, 2008).

Other authors claim that better policy was the main source of the Great Moderation. In particular, Clarida et al. (2000) argue that the adoption of an interest rate rule in 1979 that puts a sufficiently large weight on inflation, and thereby satisfies the Taylor Principle, has eliminated self-fulfilling expectations which gave rise to instability before the 1980s. More recently, Boivin and Giannoni (2006) and Leduc et al. (2007) provide similar evidence using different empirical methods and data sets. Sims and Zha (2006) and Smets and Wouters (2007), in contrast, find that changes in systematic monetary policy were only modest. They conclude that it is mostly the variance of shocks which has declined. Galí and Gambetti (2009) and Canova (2009) argue that the combination of smaller shock variances and a more stabilizing policy is consistent with the lower volatility of macroeconomic variables since the mid 1980s.\footnote{See Giannone et al. (2008) and the references therein for a summary of the empirical literature.}

Orphanides (2004) estimates an interest rate rule using real time data and finds, in contrast to Clarida et al. (2000), that monetary policy has satisfied the Taylor principle before as well as after 1979. Thus, according to these results, self-fulfilling revisions in inflation expectations were not a source of the macroeconomic instability which characterized the US economy before the 1980s. However, he proposes an alternative explanation for the higher volatility prior to the 1980s, namely a relatively strong response of the Federal Reserve to the output gap in the presence of severe measurement error. That is, by responding strongly to noisy real time estimates of the output gap, the Federal Reserve amplified the noise in the data and thereby generated output fluctuations. During the Volcker-Greenspan era, monetary policy was less activist with respect to output stabilization resulting in smoother business cycles and ultimately the Great Moderation.

The goal of this paper is to quantitatively evaluate this hypothesis. To do so, we simulate...
the effects of interest rate rules putting different weights on output stabilization using a New Keynesian model (Woodford, 2003; Clarida et al., 1999) augmented with sticky wages and habits and assume that the central bank observes only noisy estimates of the output gap. Our analysis is closely related to Orphanides (2003) who shows how an overly activist policy can increase business cycle volatility when the output gap and inflation are measured with noise. The novel feature of our paper is to quantitatively investigate this mechanism as an explanation of the Great Moderation.

Based on our simulation results, we conclude that the regime switch in U.S. monetary policy in 1979 to a less activist interest rate rule can account for up to 50 percent of the lower standard deviation of detrended real GDP which we observe since the mid 1980s. The high serial correlation of output gap noise estimated by Orphanides (2003) implies that noise shocks have been a quantitatively important source of macroeconomic fluctuations. Therefore, a less activist policy in the post-1979 period substantially dampened the transmission of these shocks relative to the pre-1979 policy regime.

In addition, we explicitly recognize that interest rate shocks are closely related to noise shocks. Empirically, it appears that the magnitude as well as the impact of interest rate shocks have declined over time. Our simulations are consistent with these empirical results. However, according to our interpretation, interest rate shocks do not necessarily represent exogenous fluctuations in monetary policy, but result to some degree from the interaction between noisy real time data and systematic monetary policy.

Thus, we argue that although a lower variance of interest rate shocks coincides with the decline in output volatility, the ultimate source of the smaller disturbances to short-term interest rates is the regime-switch in systematic monetary policy in 1979. In this sense, noise shocks, as an explanation for the Great Moderation, share some similarities with the argument in Giannone et al. (2008) which states that even if a change in the magnitude of the shocks hitting the economy appears to be a source of smoother fluctuations, the greater stability may actually be the result of a change in the propagation mechanism. Our results are also consistent with Sims and Zha (2006) who find that it was mostly the variance of policy shocks which declined whereas systematic monetary policy was subject to small variations only. We find that even a small change in the parameters of the interest rate rule may give rise to what appears to be a large reduction in the magnitude of policy shocks.

The remainder of the paper is structured as follows: Section 2 highlights the close connection between noisy data, activist policy and interest shocks. Section 3 outlines the model which forms
the basis for our analysis and Section 4 presents our main simulation results. A sensitivity analysis is provided in Section 5, while Section 6 studies the implications of a change in the stochastic process describing output gap noise. Section 7 concludes the paper.

2 The Great Moderation and Interest Rate Shocks

The purpose of this section is threefold: First, we present descriptive statistics for U.S. GDP and inflation to illustrate the magnitude of the reduction in volatility since the mid 1980s. Second, we highlight the close connection between noise shocks and interest rate shocks. And third, we estimate a vector autoregression (VAR) model to identify reduced-form interest rate shocks. In the next section, we will then compare the quantitative implications of a New Keynesian model where the central bank can only observe noisy estimates of the output gap to the empirical quantities we present in this section.

Table 1 shows descriptive statistics for real detrended GDP, $GDP_t$, and inflation, $Inf_t$, for two subsamples. The first subsample covers 1959:1 to 1979:3 and the second subsample runs from 1985:1 to 2009:1. The starting date of the first subsample is chosen to match Boivin and Giannoni (2006). The end of the first subsample is motivated by the well documented regime change in U.S. monetary policy. Although the change in the monetary policy regime occurred abruptly, the beginning of the 1980s was characterized by substantial macroeconomic volatility. Therefore, we do not cover the early 1980s which we view largely as a transitional period and take the first quarter of 1985 as the beginning of the Great Moderation period. $GDP_t$ is obtained as the residual of a regression of the log of real GDP on a constant, a linear and a quadratic time trend. Inflation is measured as the annualized rate of change between two consecutive quarters.

We see from Table 1 that the standard deviations of GDP and inflation have declined substantially after 1985. GDP is about 44 percent less volatile and the standard deviation of inflation declined by about two thirds.

We proceed by discussing the close relationship between data noise and interest rate shocks. Suppose that the central bank conducts monetary policy according to a simple interest rate rule. That is, the central bank sets the interest rate depending on the output gap, $\hat{y}_t$, and the inflation rate, $\hat{\pi}_t$. Also suppose that the central bank cannot observe the output gap, but only a noisy

\footnote{This dating of the beginning of the Great Moderation is also consistent with evidence presented in e.g. Smith and Summers (2009).}
\footnote{The series are obtained from the Federal Reserve Bank of St. Louis Economic Data database.}
\footnote{Hatted variables are measured as percentage deviations from the steady state values.}
estimate. More specifically, the central bank observes ˜\(y_t = \hat{y}_t + \hat{z}_t\), where ˜\(z_t\) denotes output gap noise. Thus, monetary policy can be described by the rule

\[
\hat{R}_t = \kappa_\pi \hat{\pi}_t + \kappa_y \hat{y}_t + \epsilon_t,
\]

where \(\kappa_\pi, \kappa_y\) characterize the responses of the interest rate to fluctuations in inflation and the noisy estimate of the output gap and \(\epsilon_t\) is a monetary policy shock. Although we will consider more general interest rate rules in the next section, the rule (1) is sufficient to highlight the main intuition.

Using the definition of ˜\(y\), the interest rate rule (1) can be rewritten as

\[
\hat{R}_t = \kappa_\pi \hat{\pi}_t + \kappa_y \hat{y}_t + \hat{\epsilon}_t,
\]

where \(\hat{\epsilon}_t = \kappa_y \hat{z}_t + \epsilon_t\). The crucial point is that, in this formulation, \(\hat{\epsilon}_t\) is a convolution of output gap noise, the output coefficient in the interest rate rule and the actual policy shock. Nevertheless, it closely resembles a shock to the interest rate equation analyzed in the large literature using monetary vector-autoregression (VAR) models (see e.g. Christiano et al., 1999). That is, when estimating a VAR with data which may not have been available in real time, the identified policy shock will also mirror data noise and systematic policy to some extent.

To get some impression about the quantitative magnitudes, we now estimate the interest rate shocks using a VAR. Since the shock to the interest rate equation of the VAR may pick up the effects of data noise and its interaction with monetary policy, we are not able to identify the actual noise shock. Nevertheless, this exercise still allows us to quantify how the overall magnitude of interest rate shocks has changed over time and therefore presents another dimension of the data to which we can compare our simulation results in the next section.

As in Boivin and Giannoni (2006) we estimate the VAR with \(GDP_t, Inf_t\) and the Federal Funds rate, \(FF_t\):

\[
Z_t = A_0 + A(L)Z_{t-1} + u_t,
\]

where \(Z_t = (GDP_t, Inf_t, FF_t)'\). This ordering of the variables is based on the standard assumption in the monetary VAR literature that monetary policy responds contemporaneously to output and inflation, whereas output and inflation respond only with a one period lag to interest rate innovations.

Figure (1) shows the impulse responses for the two subsamples. The magnitudes of the responses of output and inflation to interest rate shocks are generally more pronounced in the first subsample (see also Boivin and Giannoni, 2006). The variance decomposition, displayed
in Table 3 also suggests that the effects of interest rate shocks in the second subsample are substantially reduced. Figure (2) shows the orthogonalized interest rate shocks obtained for the two subsamples and indicates that the volatility of the shock declined over time. More specifically, we see from Table 2 that the standard deviation of the interest rate shock declined by about 50 percent as compared to the first subsample. Thus, interest rate shocks, which to some extent may mirror the effects of data noise, turn out to be of smaller magnitude and have a smaller impact on macroeconomic variables since the start of the Great Moderation.

To summarize, after 1985 macroeconomic volatility declined substantially. At the same time, the magnitude and the effects of interest rate shocks have become less pronounced. In the remainder of this paper we will evaluate the hypothesis that the change in the behavior of the Federal Reserve in the face of noisy data has resulted in smaller interest rate shocks and ultimately lower macroeconomic volatility. To do so, we calibrate and simulate a New Keynesian business cycle model and compare the simulation results under different interest rate rules to the empirical findings presented in this section.

3 Model and Calibration

In this section we describe the structure and calibration of a small-scale New Keynesian model which forms the basis for our simulations. As it is standard in the literature, we assume that households supply labor on a monopolistically competitive labor market. Household $j$ maximizes $E_0 \sum_0^\infty \beta^t \left( (c_{jt} - hc_{jt-1})^{1-\sigma} / (1 - \sigma) - n_{jt}^{1+\nu} / (1 + \nu) \right)$, where $c_{jt}$ and $n_{jt}$ denote consumption and hours worked in period $t$, $\sigma$ is the coefficient of relative risk aversion and $h$ denotes the degree of external habit formation. We denote the inverse of the labor supply elasticity by $\nu$.

Households supply differentiated types of labor $n_{jt}$ and choose their wage to maximize lifetime utility subject to a downward sloping demand equation $n_{jt}^d = (w_{jt}/w_t)^{-\phi} n_t$ for their type of labor. The consumption allocation has to satisfy: $w_{jt}n_{jt} + b_{jt-1}R_t = p_t c_{jt} + b_{jt} + div_{jt}$, where $w_{jt}$ is the nominal wage and $b_{jt-1}$ denotes nominal bond holdings that household $j$ carries over from period $t-1$, which earn a gross, nominal interest rate of $R_t$. The nominal profits redeemed to household $j$ are denoted by $div_{jt}$. As households have access to state contingent securities, they can insure against variations in household-specific labor income and therefore households are homogeneous with respect to asset holdings and consumption: $b_{jt} = b_t$ and $c_t = c_{jt}$. As in Erceg et al. (2000) we assume a Calvo wage setting scheme and include wage indexation to past inflation rates. $(1 - \theta_w)$ denotes the fraction of households that reset their wages each quarter.
The degree of wage indexation is measured by $\omega_w$.

Firms operate under monopolistic competition and each firm $i$ hires labor, $n_{it}$, and produces a differentiated good according to: $y_{it} = n_{it}$. Each firm sells its output at a price $p_{it}$ and faces the demand curve $y_{it}^d = (p_{it}/p_t)^{-\gamma} y_t$, where $p_t$ and $y_t$ denote the aggregate price level and aggregate output. As in Calvo (1983), each period, a fraction $(1 - \theta_p)$ of firms is able to adjust its price. As an additional feature we include price indexation $\omega_p$.

We assume that the decisions of firms at time $t$ are predetermined, that is, output, wages and inflation are prevented from responding contemporaneously to shocks. This assumption is consistent with the restrictions used to identify the interest rate shock in Section 2.

The intertemporal optimality condition for the household’s choice problem and goods market clearing give rise to the forward-looking IS relationship:\(^4\)

$$\hat{y}_t = \frac{1}{1 + h} E_{t-1} \hat{y}_{t+1} + \frac{h}{1 + h} \hat{y}_{t-1} - \frac{1 - h}{(1 + h)\sigma} E_{t-1}(\hat{R}_t - \hat{\pi}_{t+1}),$$

(4)

where hatted variables denote percentage deviations from the steady state. Based on the price-setting behavior of firms, we obtain the New Keynesian Phillips Curve:

$$\hat{\pi}_t = \gamma_f E_{t-1} \hat{\pi}_{t+1} + \gamma_b \hat{\pi}_{t-1} + \kappa_p E_{t-1} \hat{\varphi}_t,$$

(5)

where $\gamma_f = (\beta \theta_p)/[\theta_p + \omega_p(1 - \theta_p(1 - \beta))]$, $\gamma_b = \omega_p/[\theta_p + \omega_p(1 - \theta_p(1 - \beta))]$ and $\kappa_p = [(1 - \theta_p)(1 - \beta \theta_p)(1 - \omega_p)]/[\theta_p + \omega_p(1 - \theta_p(1 - \beta))]$. Real marginal cost are defined as $\hat{\varphi}_t = \hat{w}_t - \hat{p}_t$.

The evolution of nominal wage setting reads:

$$\Delta \hat{w}_t = \beta \chi_1 E_{t-1} \Delta \hat{w}_{t+1} + \omega_w \chi_1 \Delta \hat{\pi}_{t-1} - \beta \theta_w \chi_2 E_{t-1} \hat{\pi}_t + \chi_2 \hat{\pi}_{t-1}$$

$$+ \kappa_w E_{t-1} [\hat{mrs}_t - (\hat{w}_t - \hat{p}_t)],$$

(6)

where $\chi_1 = \theta_w/[\omega_w + \theta_w(1 - \omega(1 - \beta \theta_w))]$, $\chi_2 = (\omega_w(1 - \theta_w))/[\omega_w + \theta_w(1 - \omega(1 - \beta \theta_w))]$ and $\kappa_w = [(1 - \theta_w)(1 - \beta \theta_w)(1 - \omega_w)]/[\omega_w + \theta_w(1 - \omega(1 - \beta \theta_w))(1 + \nu \phi)]$. The marginal rate of substitution is $\hat{mrs}_t = [((1 - h)\nu + \sigma)/(1 - h)] \hat{c}_t - [\sigma h/(1 - h)] \hat{c}_{t-1}$.

To close the model, we assume that monetary policy follows an interest rate rule. As discussed in Section 2, we assume that the central bank can only observe a noisy estimate of the output gap when setting interest rates: $\hat{y}_t = \hat{y}_t + \hat{z}_t$, where $\hat{y}_t$ is the true output gap and $\hat{z}_t$ is output gap noise. Output gap noise follows an autoregressive process:

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \epsilon_{zt},$$

(7)

\(^4\)For a detailed derivation see for instance Woodford (2003).
where $\epsilon_{zt} \sim N(0, \sigma_z)$ is an i.i.d. noise shock. Specifically, monetary policy follows a rule of the type estimated by Orphanides (2004):

$$\ddot{R}_t = \rho_1 \ddot{R}_{t-1} + \rho_2 \ddot{R}_{t-2} + (1 - \rho_1 - \rho_2) \left[ \kappa_{\pi} E_t \frac{1}{1+i} (\hat{\pi}_t + \ldots + \hat{\pi}_{t+i}) + \kappa_y \tilde{y}_t \right],$$

(8)

where $\rho_1$ and $\rho_2$ determine policy inertia, $\kappa_\pi$ and $\kappa_y$ are the weights the central bank attaches to expected average inflation and output, respectively, and $i$ indicates the number of periods over which the expected average inflation rate is calculated. The specification with $i = 1$ corresponds to the case where the central bank responds only to a weighted average of the current quarterly inflation rate and the inflation rate in the next period. For higher values of $i$ the central bank behaves increasingly forward looking. Orphanides (2004) reports coefficient estimates for $i = 1, \ldots, 4$.

Note that in this setup, households and firms observe the true output gap, whereas the central bank only observes a noisy signal. Thus, we assume that private agents have better information concerning the state of the economy than the central bank. In our setting this assumption essentially boils down to saying that households know their consumption levels and firms observe the output they produce, whereas the central bank has to form estimates. As argued by Orphanides (2003), policymakers may try to infer the information of private agents through e.g. consumer sentiment surveys. However, since these data sources are subject to noise, it appears plausible that private agents are better informed than policy makers.

Our evaluation of monetary policy before and after the onset of the Great Moderation will be based on the model summarized by equations (4) - (8). To study the quantitative implications of shocks to output gap noise under different interest rate rules, we calibrate and simulate the model. The calibration is largely standard and described in detail in Table 4. To calibrate the Taylor rule and the noise process, we use the parameter estimates reported in Orphanides (2004). Table 5 summarizes the parameters of the interest rate rule for easy reference.

### 4 Activist Policy and the Great Moderation

In this section, we quantitatively explore the hypothesis that a less activist monetary policy in the presence of data noise is partly the source of the Great Moderation. Our analysis is based

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5It would be easy to consider inflation noise in addition to output gap noise. However, doing so would have very limited implications since measurement errors have been less pronounced in the inflation rate and, even more importantly, substantially less persistent according to Orphanides (2003). As we will show in Section 5 below, it is to a large extent the persistence of noise shocks which drives the results.
on counterfactual simulation exercises in which we examine to what extent the switch in the interest rate rule in 1979 reduces macroeconomic volatility keeping all other structural as well as stochastic characteristics of the model economy constant.

More specifically, we conduct the following experiment: We simulate the model and compute asymptotic moments based on the interest rate rule which describes U.S. monetary policy before 1979 according to Orphanides (2003). The outcome of this experiment is a pair of standard deviations for the inflation rate, $\sigma(\hat{\pi})$, and the output gap, $\sigma(\hat{y})$. Next, we simulate the model but re-parameterize the interest rate rule according to the estimates provided by Orphanides (2003) for the post-1979 period. By comparing the simulated standard deviations across these two simulations we are able to quantify the influence of the parameterization of the interest rate rule on the volatility of the model economy under noisy data.

Table 6 reports the outcome of this exercise for forecast horizons for the inflation rate ranging from $i = 1$ to $i = 4$ periods. The second and the third columns show the standard deviations of output and inflation under the pre-1979 interest rate rule. The next two columns report the standard deviations when we calibrate the interest rate rule according to empirical estimates for the period starting in 1979. To isolate the effect of the output coefficient, we report in the last two columns the standard deviations when we set the parameters of the interest rate rule to their pre-1979 values, except for $\kappa_y$, which we set to its post-1979 value.

The table shows that regardless of the forecast horizon, output and inflation are both roughly 40 percent less volatile under the post-1979 rule, where the volatility reduction is slightly more pronounced for the inflation rate. For longer forecast horizons, that is, higher values for $i$, the magnitudes of the volatility reductions generally increase. Comparing these results to the values for $\sigma(\hat{y})$ and $\sigma(\hat{\pi})$ reported in the last two columns of Table 6 where only $\kappa_y$ is set to the post-1979 level, we see that most of the volatility reduction in the output gap, and to a lesser extent also in the inflation rate, is due to the lower coefficient on the output gap in the interest rate rule.

Thus, a less activist interest rate rule results in sizable reductions in the volatility of economic activity. However, the question remains, how much of the lower volatility associated with the Great Moderation can be accounted for by the switch in the policy rule. Note that in our simulations, the noise shock is the only source of volatility. Since we do not explicitly model any other shocks, a comparison of our simulation results with historical data is not straightforward, since the U.S. economy was presumably hit by substantial shocks which were not related to data noise before and also after 1979. In fact, output and inflation are substantially less volatile in
the model than in the data as a comparison of the standard deviations in Tables 1 and 6 shows, which indicates that the model cannot fully account for the overall level of macroeconomic volatility.

Therefore, to quantify the degree to which the switch in the interest rate rule can account for the Great Moderation, we report in Table 7 the volatility reduction we obtain in our simulations relative to the difference between the standard deviations of output and inflation before and during the Great Moderation in the data. Recall from the previous section that the standard deviation of output dropped from 3.29 to 1.84. Thus, we observe a change of $\Delta \sigma_{GDP} = 1.45$ in the data. Similarly the standard deviation of the inflation rate dropped from 2.75 to 0.97 which corresponds to a change of $\Delta \sigma_{INF} = 1.78$ in the data. To see for how much of $\Delta \sigma_{GDP}$ and $\Delta \sigma_{INF}$, the switch to a less activist policy can account, we report in Table 7 the changes in the standard deviations $\sigma(\hat{y})$ and $\sigma(\hat{\pi})$ which we obtain with the pre- and post 1979 interest rate rule coefficients relative to $\Delta \sigma_{GDP}$ and $\Delta \sigma_{INF}$ respectively.

Table 7 shows that the switch in monetary policy in 1979 accounts for 36 percent to 58 percent of the lower output volatility associated with the Great Moderation, depending on the forecast horizon. For the inflation rate, the switch in the interest rate rule can explain 39 percent to 56 percent of the reduction in the standard deviation. The last two columns of Table 7 show the relative volatility reduction when we simulate the model with the pre-1979 interest rate rule except for $\kappa_{\pi}$ which is set to the post-1979 level. We see that changing only the coefficient on the output gap can still account for 31 percent to 48 percent of the reduction in output volatility associated with the Great Moderation and for 29 percent to 44 percent of the reduction in the standard deviation of inflation. We conclude that most of the change in macroeconomic volatility that can be attributed to the switch in systematic monetary policy, is actually due to the change in the output gap coefficient.

In short, we see that although the magnitudes of the volatility reductions depend somewhat on the exact specification of the interest rate rule, these reductions are non-negligible in all cases. In this sense, the regime switch in U.S. monetary policy to a less activist policy in 1979 may indeed have contributed substantially to the reduction in macroeconomic volatility which characterizes the Great Moderation period.

Figure 3 shows how the variables in the model respond to a one unit shock in output gap noise when the central bank conducts monetary policy with a forecast horizon of $i = 4$ period.\textsuperscript{6} The sub-figures in the left column show the results for the pre-1979 calibration and the sub-figures

\textsuperscript{6}Shorter forecast horizons produce largely similar results.
in the right column are based on the post-1979 calibration.

Qualitatively we obtain the same picture with both calibrations. Since the central bank interprets the positive innovation to output gap noise as an increase in the output gap, it tightens monetary policy. Consequently, output gap and inflation decline. The output gap reaches its trough after five quarters and returns subsequently to its baseline level. Due to the assumption of sticky prices and wages the inflation rate slowly falls along a hump shaped path and reaches its peak response 10 quarters after the initial shock. The interest rate initially increases but turns negative after seven quarters and eventually returns to its baseline level.

Although, the qualitative patterns are the same, the magnitudes of the responses differ substantially for the two calibrations. Under the post-1979 calibration, the maximum responses of the output gap and inflation are less than half of what we obtain with the pre-1979 calibration.

The reason behind these quantitative differences is the strong amplification of output gap noise via the interest rate rule in the pre-1979 calibration. Analogous to the discussion in Section 2, the interest rate rule in the model can be written as

\[
\hat{R}_t = \rho_1 \hat{R}_{t-1} + \rho_2 \hat{R}_{t-2} + (1 - \rho_1 - \rho_2) \left[ \kappa_\pi E_t \frac{1}{1 + \kappa_\pi} (\hat{\pi}_t + \ldots + \hat{\pi}_{t+i}) + \kappa_y \hat{y}_t \right] + \hat{\varepsilon}_t,
\]

where \(\hat{\varepsilon}_t = (1 - \rho_1 - \rho_2) \kappa_y \hat{z}_t\) closely resembles an interest rate shock, although it is ultimately data noise which drives the dynamics of the model. From the last row of Figure 3 we see that \(\hat{\varepsilon}_t\) responds substantially less to noise shocks in the post-1979 calibration. Thus, while output gap noise plays only a limited role in the post-1979 calibration, it gives rise to what appear to be substantial unsystematic changes in monetary policy in the pre-1979 calibration. This result is consistent with the empirical observation that interest rate shocks have been less pronounced since the onset of the Great Moderation.\(^7\) Overall, our simulations show that a switch in the systematic part of monetary policy implies a decline of the magnitude of interest rate fluctuations, which is consistent with the empirical impulse responses plotted in Figure 1.

In short, the interaction between systematic monetary policy and noisy data gives rise to aggregate volatility if the central bank pursues an activist output stabilization policy. The switch in the interest rate rule that characterized U.S. monetary policy according to Orphanides (2004) and the resulting lower amplification of noise shocks are consistent with empirical evidence concerning the size and impact of interest rate shocks.

\(^7\)When we simulate the model with the pre-1979 and post-1979 interest rate rules, we find that the standard deviation of \(\hat{\varepsilon}_t\) declines between 71 and 77 percent depending on the forecast horizon \(i\).
5 Sensitivity Analysis

Now we dig a little deeper and analyze how our simulation results are linked to the calibration. In particular, we explore how $\kappa_y$, $\kappa_\pi$, $\rho_1$, and $\rho_z$ influence macroeconomic volatility in the model. For each of these parameters, Figure 4 shows how the simulated standard deviations of the output gap $\sigma(\hat{y})$ and the inflation rate $\sigma(\hat{\pi})$ change when we vary the parameter under consideration, conditioned on all other parameters set to their pre-1979 values. The subplots in the first row of Figure 4 show the effects of $\kappa_y$ and $\kappa_\pi$ and the second row of the figure shows how $\rho_1$ and $\rho_z$ influence volatility.

We see that a higher value for the output gap coefficient, $\kappa_y$, which we conjecture to be a driving force of the Great Moderation, substantially increases the volatility of fluctuations in the output gap and in the inflation rate. In particular the figure reveals that if monetary policy had neglected the output gap, noisy data would have had essentially no impact on the economy. The more activist monetary policy becomes, defined by increasing values of $\kappa_y$, the more volatility is imputed into the economy. According to the figure, $\kappa_y = 1$ generates standard deviations between $\sigma_y = 1.54$ for a forecast horizon of $i = 1$ up to $\sigma_y = 1.57$ for $i = 4$. Accordingly, a reduction of the output coefficient from $\kappa_y = 1$ to $\kappa_y = 0$, while keeping all other parameter constant at their pre-1979 levels, would be able to explain the complete decline in volatility found in the data which is round about $\Delta \sigma_y = 1.45$ and would therefore fully account for the Great Moderation.

An increase in the interest rate rule coefficient on inflation, $\kappa_\pi$, attenuates the adverse effects of noisy information on volatility. Intuitively, after a positive noise shock the central bank increases the nominal interest rate which leads to a negative output gap and a decline of the inflation rate. If the central bank is more responsive to fluctuations in the inflation rate, then monetary policy quickly counteracts these fluctuations induced by noise shocks. Although quantitatively, $\kappa_y$ exerts a stronger influence than $\kappa_\pi$, a stronger response to inflationary pressure still helps to stabilize the economy to a non-negligible extent. In this sense, our results are also in line with Clarida et al. (2000) who argue that a stronger reaction of the central bank to inflationary pressure gave rise to greater stability in the U.S. economy. However, their argument is based on the elimination of sunspot fluctuations, whereas we focus on the transmission of a particular fundamental shock.

If output gap noise becomes more persistent as measured by $\rho_z$, we see that output and inflation, both become substantially more volatile. This is due to the fact that persistent noise
shocks translate into persistent fluctuations in the interest rate, which have a large impact on aggregate demand due to the forward-looking nature of the model. While the measured standard deviation of the output gap is between 0.21 to 0.25 for a coefficient of \( \rho_z = 0.5 \), it sharply increases to 0.62 to 0.70 for \( \rho_z = 0.8 \). Thus, Figure 4 indicates that our success to explain the Great Moderation by a less activist policy is strongly driven by the high serial correlation in output gap noise.

Next, we turn to the influence of policy inertia, \( \rho_1 \). Figure 4 shows that a more inertial monetary policy, that is, higher \( \rho_1 \), increases output volatility. For inflation volatility, the increase is less pronounced for moderate degrees of policy inertia as the Phillips curve is relatively flat. Intuitively, high inertia implies that noise shocks have a long-lasting influence on the interest rate, despite the fact that the initial response is dampened. In this sense, high policy inertia additionally increases the persistence of the effect of noise shocks.

Note that Figure 4 also highlights that qualitatively, our results are invariant with respect to the chosen forecast horizon. However, we also see that increasing the forecast horizon from \( i = 1 \) to \( i = 4 \) generally increases the standard deviations. This finding is a natural extension of our previous results. Given that the inflation rate is a mean reverting process the central bank gets effectively less responsive to the inflation rate as the weighted average over a longer forecast horizon of a mean reverting process is likely to be smaller in value than the weighted average over a shorter forecast horizon. Thus, reacting to forecasts of the inflation rate operates in a similar way as a reduction of the inflation coefficient \( \kappa_\pi \).

Overall, we see that while the extent to which the central bank reacts to inflationary developments, \( \kappa_\pi \), and the degree of policy inertia, \( \rho_1 \), have some influence on macroeconomic volatility in the presence of noisy data, the strength of the feedback from the output gap to monetary policy, as measured by \( \kappa_y \), and the serial correlation of output gap noise, \( \rho_z \), are the parameters that primarily drive our results.

6 Did a Reduction in Output Gap Noise Contribute to the Great Moderation?

So far, our simulations were based on the assumption that the stochastic process governing output gap noise was stable over time. Clearly, this need not be the case. In fact, it appears plausible that in addition to the switch in systematic policy, the time series properties of output noise.
gap noise have also changed. In particular, output gap noise may have declined along with the
reduction in macroeconomic volatility. In this case our simulations would overestimate the effect
of the switch to a less activist policy.

In this section we relax the assumption of stable process for output gap noise. We proceed
in two steps: First, we construct estimates of historical real time output gaps and characterize
the time series properties of the resulting output gap noise for the samples 1965Q3 - 1979Q3
and 1985Q3 - 2009Q1.

To construct proxies for output gap noise we use the real time GDP data provided by the
Federal Reserve Bank of Philadelphia. The data set consists of different vintages of GDP series
which were available in real time ranging from 1965Q3 to 2009Q1. All vintage series start in
1947:1. To obtain an estimate of the real time output gap in 1965Q3, we detrend the GDP series
which has been available in 1965Q3 and calculate the real time output gap as the deviation from
a trend. We repeat this procedure for each of the following quarters until 2009Q1 and thereby
obtain a series of estimated real time output gaps. Finally, to obtain an estimate of output gap
noise, we calculate another output gap series as the deviation of real GDP from trend based on
the final, revised GDP series. The difference between the two series is our measure for output
gap noise. We use two alternative detrending methods: We (i) regress the log of real-time GDP
on a constant, a time trend and the square of the time trend and (ii) use the Hodrick-Prescott
(HP) filter with a smoothing parameter of 1600.

Note that this approach is rather mechanical and does not capture any qualitative aspects,
such as judgment, that were presumably involved when the Federal Reserve generated real time
estimates of the output gap. This approach also differs from Orphanides (2004) who uses data
on the macroeconomic outlook prepared for meetings of the Federal Open Market Committee
(FOMC). Nevertheless, Table 8 shows that the time series properties of our mechanically esti-
mated output gap noise series are remarkable similar to the ones presented in Orphanides (2004)
and summarized in Table 4. This is true for both, the quadratic trend as well as the HP filter.
Although the standard deviation, \( \sigma_z \), of our measures of output gap noise is somewhat lower
than in Orphanides (2004) we also find that output gap noise is highly persistent as indicated
by the high values of \( \rho_z \).

Comparing the properties of the noise processes across the subsamples we see that the average
size of noise shocks, as measured by \( \sigma_z \), has declined substantially, as expected. We also see,
however, that \( \rho_z \) has remained high or has even slightly increased. Thus, even though noise
shocks have become smaller, they remain highly persistent.
To assess how the change in the stochastic process for output gap noise may have interacted with the switch in the interest rate rule, we now repeat our simulations. In particular, we now change the parameters governing the noise process along with the interest rate rule coefficients. To calibrate the noise process, we use the results obtained using the HP Filter reported in the last column of Table 8.

Table 9 shows that the simulation results closely resemble those presented in Table 6 in a qualitative sense. However, due to the lower standard deviation of noise shocks in this calibration, \( \sigma(\hat{y}_t) \) and \( \sigma(\hat{\pi}_t) \) turn out to be lower, regardless of the forecast horizon \( i \). This is true for the pre-1979 and also for the post-1979 calibration of the interest rate rule. Comparing the volatilities under the pre-1979 and post-1979 interest rate rules shows that we now obtain somewhat larger volatility reduction, which is due to the reduction of the standard deviation of noise shocks that occurred along with the switch in the interest rate rule.

Nevertheless, the volatility reductions in output and inflation volatility are are still substantially driven by the lower value of \( \kappa_y \). This can be seen from the last two columns which show the results, when we change only \( \kappa_y \) to its post-1979 value and calibrate the remaining parameters including the process for output gap noise to match characteristics of the pre-1979 sample. Thus, even if we explicitly take into account that noise shocks have become less pronounced over time, we find that the switch in the interest rate rule remains a quantitatively important source of the lower macroeconomic volatility.

Turning to Table 10, where we compare the volatility reductions in the simulated series to what we observe in the data, we see that the switch in the interest rate together with the change in the process governing output gap noise accounts for 46 to 62 percent of the reduction in output volatility and for 28 to slightly below 44 percent of the lower inflation volatility associated with the Great Moderation. Hence, taking the reduction in the magnitude of noise shocks into account increases the fraction of volatility which our simulations can account for.

To isolate the effect of a less activist policy, we simulate the model with the pre-1979 calibration of the noise process and the interest rate rule, except for \( \kappa_y \), which we set to its post-1979 value. The last two columns show the results. We see that the lower value of \( \kappa_y \) can account for a reduction of output volatility ranging from 29 to 49 percent of what we observe in the data. Concerning inflation volatility, keeping everything constant except for \( \kappa_y \) results in volatility reductions ranging from around 22 percent to 30 percent of the more stable inflation rate associated with the Great Moderation period.

Thus, we conclude that although a decline in the standard deviation of noise shocks also
contributed to the Great Moderation, the switch to a less activist policy turns out to be quantitatively more important.

7 Conclusions

In this paper we quantitatively evaluate the role of a less activist monetary policy in the presence of data noise as an explanation for the Great Moderation. Smets and Wouters (2007) state that whether policy has contributed to the reduction of shocks and consequently to the Great Moderation remains an interesting research question. Our simulations suggest that the answer to this question is affirmative to a large extent. We argue that interest rate fluctuations partly mirror the response of monetary policy to noise shocks and therefore a less activist policy dampens the transmission of these shocks substantially.

Based on a calibrated New Keynesian model, we find that the switch to an interest rate rule in 1979 that reacts less to fluctuations in the output gap can account for up to half of the lower standard deviation of real GDP which we observe since the mid 1980s. Essentially, the high serial correlation of output gap noise implies that noise shocks are a quantitatively important source of macroeconomic fluctuations. Our results are also consistent with the empirical observation that interest rate shocks have been less pronounced since the beginning of the Great Moderation period and that the impact of interest rate shocks on output and inflation has been dampened.

In our analysis, we highlight the interaction between shocks to output gap noise and systematic monetary policy. In this sense we contribute to the literature that emphasizes the role of exogenous shocks as well as policy as the source of the Great Moderation. According to this branch of the literature, the question is not primarily whether better policy or good luck, in the form of smaller shocks, is the ultimate source of the Great Moderation, but how policy and exogenous shocks together resulted in greater macroeconomic stability. In this paper we specifically focus on the interaction between noisy data and monetary policy. Nevertheless, it remains an interesting topic for future research to identify and explore additional aspects where changes in the structure of the economy interacted with exogenous shocks in such a way to result in smoother business cycles.

References


Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>1959Q1 -1979Q3</th>
<th>1985Q1 -2009Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP Mean</td>
<td>0.84</td>
<td>0.88</td>
</tr>
<tr>
<td>Inf Mean</td>
<td>4.29</td>
<td>2.46</td>
</tr>
<tr>
<td>Max</td>
<td>5.87</td>
<td>4.18</td>
</tr>
<tr>
<td>Min</td>
<td>-6.74</td>
<td>-4.96</td>
</tr>
<tr>
<td>Stdv</td>
<td>3.29</td>
<td>1.84</td>
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<tr>
<td>Obs</td>
<td>83</td>
<td>97</td>
</tr>
</tbody>
</table>

Table 2: Empirical Interest Shocks

<table>
<thead>
<tr>
<th></th>
<th>1959Q1 - 1979Q3</th>
<th>1985Q1 - 2009Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Max</td>
<td>1.92</td>
<td>0.32</td>
</tr>
<tr>
<td>Min</td>
<td>-1.15</td>
<td>-0.62</td>
</tr>
<tr>
<td>Stdv</td>
<td>0.48</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 3: Variance Decomposition

<table>
<thead>
<tr>
<th>Horizion</th>
<th>1959Q1 -1979Q3</th>
<th>1985Q1 -2009Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP Inf FF</td>
<td>GDP Inf FF</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>19.81</td>
<td>34.06</td>
</tr>
<tr>
<td>8</td>
<td>55.48</td>
<td>36.60</td>
</tr>
<tr>
<td>12</td>
<td>64.07</td>
<td>39.87</td>
</tr>
<tr>
<td>16</td>
<td>65.21</td>
<td>42.25</td>
</tr>
<tr>
<td>Parameter</td>
<td>Calibration</td>
<td></td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion</td>
<td>$\sigma$</td>
<td>1.62</td>
</tr>
<tr>
<td>Inverse of the labor supply elasticity</td>
<td>$\nu$</td>
<td>2.45</td>
</tr>
<tr>
<td>Calvo prices</td>
<td>$\theta_p$</td>
<td>0.87</td>
</tr>
<tr>
<td>Serial correlation of output gap noise</td>
<td>$\rho_z$</td>
<td>0.911</td>
</tr>
<tr>
<td>Stdv. of measurement noise</td>
<td>$\sigma_z$</td>
<td>0.93</td>
</tr>
<tr>
<td>Monopoly power of households</td>
<td>$\phi$</td>
<td>5.00</td>
</tr>
<tr>
<td>Calvo wages</td>
<td>$\theta_w$</td>
<td>0.80</td>
</tr>
<tr>
<td>Habit formation</td>
<td>$h$</td>
<td>0.69</td>
</tr>
<tr>
<td>Wage indexation</td>
<td>$\omega_w$</td>
<td>0.64</td>
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<tr>
<td>Price indexation</td>
<td>$\omega_p$</td>
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Table 5: Calibration of the Interest Rate Rule

<table>
<thead>
<tr>
<th></th>
<th>$i = 1$</th>
<th>$i = 2$</th>
<th>$i = 3$</th>
<th>$i = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_\pi$</td>
<td>1.49</td>
<td>1.89</td>
<td>1.54</td>
<td>1.97</td>
</tr>
<tr>
<td>$\kappa_y$</td>
<td>0.46</td>
<td>0.18</td>
<td>0.53</td>
<td>0.18</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.94</td>
<td>0.85</td>
<td>0.87</td>
<td>0.82</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>-0.26</td>
<td>-0.08</td>
<td>-0.21</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Notes: The table shows the calibration of the interest rate rule coefficients for forecast horizons $i = 1, \ldots, 4$ as reported in Orphanides (2004), p. 161. Columns labeled 'pre-1979' show coefficients estimated for the pre-1979 regime and and Columns labeled 'post-1979' show the coefficients for the post-1979 regime.
Table 6: Macroeconomic Volatility

<table>
<thead>
<tr>
<th>Forecast Horizon</th>
<th>pre-1979 rule</th>
<th>post-1979 rule</th>
<th>pre-1979 rule except κ_y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>σ(\hat{y})</td>
<td>σ(\hat{\pi})</td>
<td>σ(\hat{y})</td>
</tr>
<tr>
<td>i = 1</td>
<td>1.20</td>
<td>1.20</td>
<td>0.67</td>
</tr>
<tr>
<td>i = 2</td>
<td>1.24</td>
<td>1.27</td>
<td>0.64</td>
</tr>
<tr>
<td>i = 3</td>
<td>1.23</td>
<td>1.27</td>
<td>0.55</td>
</tr>
<tr>
<td>i = 4</td>
<td>1.33</td>
<td>1.40</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Notes: The table reports the standard deviations of the simulated variables. Under the header 'pre-1979 rule' the table reports the standard deviations when the model is simulated with the interest rate rule in place before 1979 for forecast horizons ranging from i = 1 to i = 4. Under the header 'post-1979 rule' we report the standard deviations we obtain when we simulate the model with the interest rule that characterizes the post-1979 period. Under the header 'pre-1979 rule except κ_y' we simulate the model with the pre-1979 rule except for the output gap coefficient, which we set to its post-1979 level.

Table 7: Reduction in Volatility relative to the Data

<table>
<thead>
<tr>
<th>Forecast Horizon</th>
<th>post-1979 Rule except κ_y</th>
<th>pre-1979 rule except κ_y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>σ(\hat{y})</td>
<td>σ(\hat{\pi})</td>
</tr>
<tr>
<td>i = 1</td>
<td>36.55</td>
<td>37.08</td>
</tr>
<tr>
<td>i = 2</td>
<td>41.38</td>
<td>41.57</td>
</tr>
<tr>
<td>i = 3</td>
<td>46.70</td>
<td>45.51</td>
</tr>
<tr>
<td>i = 4</td>
<td>57.24</td>
<td>54.49</td>
</tr>
</tbody>
</table>

Notes: The table reports the volatility reduction in the model relative to the data when we simulate the model either with the post-1979 rule or with the pre-1979 rule and κ_y set to its post-1979 value.
Table 8: Output Gap Noise

<table>
<thead>
<tr>
<th></th>
<th>Quadratic Trend</th>
<th>HP-Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1965Q3 -1979Q3</td>
<td>0.94 0.90</td>
<td>0.90 0.89</td>
</tr>
<tr>
<td>1985Q1 -2009Q1</td>
<td>0.99 0.94</td>
<td>0.69 0.45</td>
</tr>
</tbody>
</table>

Notes: The table reports the estimated serial correlation, $\rho_z$, and standard deviation, $\sigma_z$, of noise shocks, based on either a quadratic trend or the HP filter.

Table 9: Macroeconomic Volatility Allowing for a Change in Output Gap Noise

<table>
<thead>
<tr>
<th>Forecast Horizon</th>
<th>pre-1979 rule</th>
<th>post-1979 rule</th>
<th>pre-1979 rule except $\kappa_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma(\hat{y})$</td>
<td>$\sigma(\hat{\pi})$</td>
<td>$\sigma(\hat{y})$</td>
</tr>
<tr>
<td>$i = 1$</td>
<td>1.06</td>
<td>0.94</td>
<td>0.39</td>
</tr>
<tr>
<td>$i = 2$</td>
<td>1.10</td>
<td>1.01</td>
<td>0.37</td>
</tr>
<tr>
<td>$i = 3$</td>
<td>1.09</td>
<td>1.01</td>
<td>0.31</td>
</tr>
<tr>
<td>$i = 4$</td>
<td>1.18</td>
<td>1.12</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Notes: The table reports the standard deviations of the simulated variables. Under the header ‘pre-1979 rule’ the table reports the standard deviations when the model is simulated with the interest rate rule in place before 1979 for forecast horizons ranging from $i = 1$ to $i = 4$. Under the header ‘post-1979 rule’ we report the standard deviations we obtain when we simulate the model with the interest rule that characterizes the post-1979 period. Under the header ‘pre-1979 rule except $\kappa_y$’ we simulate the model with the pre-1979 rule except for the output gap coefficient, which we set to its post-1979 level. The process for output gap noise is calibrated according the last column in Table 8.

Table 10: Relative Volatility When Output Gap Noise Changes

<table>
<thead>
<tr>
<th>Forecast Horizon</th>
<th>post-1979 Rule and Noise</th>
<th>pre-1979 rule except $\kappa_y$ and pre 1979 Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma(\hat{y})$</td>
<td>$\sigma(\hat{\pi})$</td>
</tr>
<tr>
<td>$i = 1$</td>
<td>46.21</td>
<td>28.09</td>
</tr>
<tr>
<td>$i = 2$</td>
<td>50.34</td>
<td>32.58</td>
</tr>
<tr>
<td>$i = 3$</td>
<td>53.79</td>
<td>36.52</td>
</tr>
<tr>
<td>$i = 4$</td>
<td>62.07</td>
<td>43.82</td>
</tr>
</tbody>
</table>

Notes: The table reports the volatility reduction in the model relative to the data when we simulate the model either with the post-1979 rule or with the pre-1979 rule and $\kappa_y$ set to its post 1979 value. The process for output gap noise is calibrated according the last column in Table 8.
Figure 1: Impulse Responses: 1959Q1 - 1979Q3 and 1985Q1 - 2009Q1

Notes: The Figure shows the responses of real, detrended GDP, $GDP_t$, the detrended inflation rate, $Inf_t$, and the Federal Funds Rate, $FF_t$ to shocks to the $FF_t$ equation along with 95 percent Hall percentile standard error bands for the two samples 1959Q1 - 1979Q3 and 1985Q1 - 2009Q1.
Figure 2: Interest Rate ($F_{F_t}$) Shocks

Notes: The Figure shows orthogonalized shocks to the $F_{F_t}$ equation for the two samples 1959Q1 - 1979Q3 and 1985Q1 - 2002Q2.
Figure 3: Impulse Responses to a Noise Shock for Different Values of $\kappa_y$

Notes: The Figure shows impulse responses for the pre-1979 value of $\kappa_y$ (left column) and the post-1979 value of $\kappa_y$ (right column) and a forecast horizon of $i = 4$ quarters.
Notes: The Figure shows how either $\sigma_y$ or $\sigma_x$ depends on the parameters of the model for different forecast horizons $i = 1, \ldots, 4$. All parameters, except for the parameter under consideration, are calibrated as in Table 4.