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Weakening the SALANT-condition for the comparison of mean durations

by

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Abstract

The classical Salant-condition for the comparison of the mean interrupted spell length of the stock of unemployed and the mean completed spell length of the corresponding flow can be substantially weakened: for a NWUE [New Worse than Used in Expectation] distribution the former is greater than the latter, and the reverse is true for a NBUE [New Better than Used in Expectation] distribution. Analogous results for the coefficient of variation and the Gini-index are established.

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1.Introduction

In the analysis of unemployment durations selecting the proper duration statistics is not a trivial problem. The two most widely used concepts are:

(i) the mean completed spell length of the flow. i.e. a cohort of people becoming unemployed at the same time, denoted by m

(ii) the mean interrupted spell length of a stock, i.e. the mean time elapsed between inception and survey date for all spells in progress at that date, denoted by b

It is well known (Salant(1977), Heckman (1987)) that the stock is a length-biased sample of the flow thus implying a longer mean duration of the former. On the other hand the statistic b does not record the full length of spells but censors them at the survey date thus causing the mean to be smaller compared to the full-length distribution. The comparative strength of the 'length-bias' vis-a-vis the 'interruption-bias' determines whether $b \geq m$ or $b \leq m$.

In a classical paper Salant (1977) has established a sufficient condition for determining which bias predominates:

(i) a decreasing failure rate (DFR) distribution of the completed spell lengths in the cohort implies $b \geq m$

(ii) an increasing failure rate (IFR) distribution of the completed spell lengths in the cohort implies $b \leq m$

(iii) the constant failure rate of the exponential distribution implies $b = m$

This condition has played a central role in the subsequent literature (e.g. Frank (1978), Beach and Kaliski (1983), OECD (1984), Jones and Riddel (1995), Corak (1996), Corak and Heisz (1996)): despite being only a sufficient condition there is a tendency to interpret the typically observed relation $b > m$ as the result of a decreasing probability to leave unemployment.

The present note shows in Section 2 that a considerably weaker sufficient condition can be established, viz. that the underlying distribution be New Worse than Used in Expectation (NWUE) or New Better than Used in Expectation (NBUE) respectively. Section 3 extends this finding to bounds for the coefficient of variation and the Gini-index.

2. A weaker condition

Define a variable X with support [0,z] and density f(.) and distribution function F(.). representing the spell length in a cohort. Assume F(0) = 0 and f(x) =F'(x) exists for all x. In a stationary environment the induced distribution with density and cumulative distribution g(.) and G(.) respectively represents the interrupted spell lengths. The relationship between these two is as follows (cf. Salant (1977)):

$$g(x) = \frac{1 - F(x)}{m} \qquad [1]$$

with

$$m = \int_0^z xf(x)dx = \int_0^z [1 - F(x)]dx \qquad [2]$$

The mean b is given by:

$$b = \int_0^z xg(x)dx = \int_0^z [1 - G(x))]dx \qquad [3]$$

Therefore

$$b \geq (\leq) m \;\Leftrightarrow\; \int_0^z [1 - \mathrm{G(x)}]\mathrm{dx} \geq (\leq) \int_0^z [1 - F(x)]dx \qquad [4]$$

[4] is certainly satisfied, if

$$\frac{1 - G(x)}{1 - F(x)} \geq (\leq) 1 \;\; \forall x \qquad [5]$$

Notice that r(x), the mean residual life at time x

$$r(x) = \int_x^z (t - x)\frac{f(t)}{1 - F(x)}dt = \int_x^z \frac{1 - F(t)}{1 - F(x)}dt \qquad [6]$$

can be re-expressed as:

$$r(x) = \frac{m[1 - G(x)]}{1 - F(x)} \qquad [6']$$

Thus condition [5] is equivalent to

$$\frac{r(x)}{m} \geq (\leq) 1 \;\; \forall \mathrm{x} \qquad [5']$$

or remembering $m = r(0)$

$$r(x) \geq (\leq) r(0) \;\; \forall x \qquad [5'']$$

[5’'] is the defining condition for the NWUE (NBUE) class of distributions (cf. Savage (1987)), characterized by the fact that the mean residual life throughout the whole process is greater (less) than the expected life at birth. Thus the following propositions can be established:

(i) if the distribution of the completed spell lengths in the cohort is NWUE: $b \geq m$

(ii) if the distribution of the completed spell lengths in the cohort is NBUE: $b \leq m$

(The exponential distribution is a member of both classes, and has $b = m$)

It can easily be shown that the DFR (IFR) class of distributions is a proper subclass of the NWUE (NBUE) class. The hazard rate h(x) is defined as:

$$h(x) = \frac{f(x)}{1-F(x)} \quad \forall x, \text{ s.t } 1\text{-}F(x) > 0 \qquad [7]$$

Therefore

$$r(x) = \int_x^z \frac{1}{h(t)} \frac{f(t)}{1-F(x)} dt \qquad [8]$$

The mean residual life is a weighted average of the inverse of the hazard-rates. Thus a DFR (IFR) distribution obviously implies

$$\frac{dr(x)}{dx} \geq (\leq) 0 \quad \forall \text{x} \qquad [9]$$

and thus

$$r(x) \geq (\leq) \, r(0) \quad \forall \text{x} \qquad [9']$$

The Salant-condition is thus a special case of the weaker proposition above.

3. Extensions

As is well known (cf. Salant (1977, p 41) the coefficient of variation V for the completed spell lengths in the cohort is related to the means by:

$$\frac{b}{m} = \frac{1}{2}(1+V^2) \qquad [10]$$

From the above reasoning it follows immediately that for a NWUE (NBUE) distribution $V \geq (\leq) 1$. This weakens the condition, given by Barlow and Proschan (1965, p.33), that a DFR (IFR) distribution implies $V \geq (\leq) 1$.

A similar result can be obtained for the Gini-index R. The definition

$$R = \frac{1}{m}\int_0^z [F(x)(1 - F(x)]dx \qquad [11]$$

can be restated as (cf. Riese (1987))

$$R = \int_0^1 [1 - G(x)]dF \qquad [11']$$

This leads to

$$\int_0^1 \{[1 - G(x)] - [1 - F(x)]\}dF = R - \frac{1}{2} = R - R_{\exp} \qquad [12]$$

where $R_{\exp}$ is the Gini-coefficient of the exponential distribution.

From [12] it is evident that a NWUE (NBUE) distribution implies $R \geq (\leq)\frac{1}{2}$. This result is noteworthy with respect to the celebrated article by Clark and Summers (1979), who - without directly using the Gini coefficient – argue that it is a decreasing hazard rate, which compared to the often used constant hazard rate of Markov models, produces a much higher concentration of unemployment. However, the much weaker requirement that the underlying distribution be NWUE would cause the same result.

4. Conclusion

We have shown that the classical sufficient condition as to the comparative strength of the 'length bias' vis-a-vis the 'interruption bias' in the measurement of unemployment durations given by Salant (1977) can be substantially weakened: Not only for the rather restrictive classes of DFR (IFR) distributions of the completed spell lengths in the flow can the relative magnitudes of b and m be computed, but for the much wider class of NWUE (NBUE) distributions. The latter classes also imply simple and intuitive bounds for the coefficient of variation and the Gini-index.

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