Abstract

External revenue such as foreign aid or external spending such as elite capture constitute important components of the government’s budget constraint in many developing countries. We derive optimal public spending – and the resulting optimal deficit and debt – in an optimal control framework when the government seeks to maximize the utility of constituents in the presence of external revenue or spending that cannot be influenced by the government directly but is contingent on public revenue and debt. We find that in this context, a policy of running budget deficits and accumulating debt becomes optimal. In simulations, we characterize the size and the path-dependency of the optimal deficit and debt.


Keywords: optimal budget deficits, optimal control, foreign aid, capture.

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1. Introduction

The development of budget deficits and public debt in developing countries is an ongoing source of concern. Unsustainable fiscal policies and the misuse of public funds are viewed as major contributors to the fact that many of the world’s poorest countries seem to be trapped in poverty. For example, public funds are sometimes appropriated by local elites for their own or their followers’ gain, a practice known as capture. On the other hand, public funds provided in the form of foreign aid are suspected to increase moral hazard and induce governments to run into even more debt.\footnote{Foreign aid and capture are examples for revenue and spending (negative revenue) components of the public budget that are external in the sense that they cannot directly be influenced by the government. Furthermore, they are often linked to the size of the budget: both can be characterized as a function of public revenue and debt, as argued below. In this paper, we show how external funds can affect countries’ optimal decisions on public spending, the deficit and debt.}

We use a dynamic optimization model to derive optimal spending decisions in the presence of external revenue, for example in the form of foreign aid. The model can equally be interpreted to apply to negative external revenue (i.e. spending), for example in the form of capture. We assume that external revenue decreases (external spending increases) in the discretionary budget, i.e. public revenue minus debt. This mirrors the fact that foreign aid is targeted primarily at poor countries with high levels of debt. Debt relief, in particular, is provided specifically to countries that exceed a certain threshold on debt in the HIPC (Heavily Indebted Poor Countries) initiative of the IMF and World Bank. Elite capture, on the other hand, can be expected to increase with an increase in net revenue. As a result, the government can increase external revenue or decrease external spending and, therefore, increase the current funds available for its constituency, by increasing the current budget deficit. In doing so, however, the government also reduces future public funds, as it incurs a higher cost of debt service. The model identifies the optimal dynamic trade-off between the current and future budgets and derives expressions for the optimal deficit and debt across time.

\footnote{See for example Easterly (2002).}
We find that it becomes optimal for the government to run a budget deficit in the presence of external funds. The optimal budget is balanced in the absence of such funds. The deficit arises because external revenue (spending) that increases (decreases) in debt alleviates the cost of debt service. Basically, part of the cost is borne by the external components of the budget. As a result, a policy of budget deficits and debt becomes optimal. Apart from the share of external funds, absolute optimal levels depend on public revenue and the interest rate as well as on the original stock of debt and on time and are, therefore, path-dependent. We show how the effect of the share of external funds on the deficit and debt changes in time. For example, the more time that has elapsed since the beginning of a given time path, the more likely it is for the optimal deficit to be decreasing in external funds.

As described below, some of the empirical studies that analyze the association between foreign aid and deficits find it to be positive, and some find it to be negative. Our results on the path-dependency of optimal spending policies can provide a theoretical argument for the variety of empirical findings on the effects of foreign aid on borrowing across countries and time. Furthermore, our results extend the literature on fiscal policy in developing countries by showing how capture can contribute to excessive spending behaviour of governments.

Foreign aid in its various forms (grants, loans, debt relief) and from various sources (multilateral development banks, bilateral, NGOs) represents a significant share of revenue in many low-income countries. For example, net official development aid (ODA)\(^2\) totalled more than 50 per cent of gross national income during 2004-2006 on average in Burundi, around 30 per cent in Afghanistan and Rwanda and around 10 per cent in Cambodia, Ghana and Zambia, to name only a few.\(^3,4\) Debt relief in particular has significantly increased in the face of a build-up of foreign debt since the late 1970s.\(^5\) In 1996, the IMF and the World Bank started the Heavily Indebted Poor Countries (HIPC) initiative and

\(^2\) ODA comprises bilateral as well as multilateral aid from the OECD’s DAC (Development Assistance Committee) member countries to Part I List of Aid Recipients (developing countries).

\(^3\) UN Statistics online, [http://www.unstat.org](http://www.unstat.org), as of July 2008.


\(^5\) Easterly (2002) gives a detailed history of debt relief by international lenders.
they extended it in 1999, with the aim of ‘ensuring that no poor country faces a debt burden it cannot manage’. The initiative allows for partial or total relief on eligible debts of poor countries that face an ‘unsustainable debt burden, beyond traditionally available debt-relief mechanisms’.

Even though these external flows are typically conditional not only on poverty and the level of debt but also on so-called good governance, many studies document that governance in the receiving countries is often not among the primary determinants of foreign aid. Perhaps not surprisingly, the success of such measures, in particular with respect to the goal of maintaining a sustainable level of debt in receiving countries, remains disputed. In fact, there is evidence that foreign aid and debt relief programs on the contrary might have contributed to an increase in poor countries’ borrowing and debt. Easterly (2002), for example, shows that debt relief for HIPC in the 1980s and 1990s was positively associated with new borrowing and draws the provoking conclusion that ‘the HIPCs got to be HIPCs in part by borrowing from the World Bank and IMF’. Further evidence on the relation between foreign aid and deficits is mixed. Feeny and McGillivray (2003), McGillivray and Ouattara (2005) on the Côte d’Ivoire and Brownbridge and Tumusiime-Mutebile (2007) on Uganda find that foreign aid increased deficits, while Ouattara (2006) and Feeny (2007) on Melanesia find that foreign aid decreased deficits in the 1980s and 1990s.

Developing countries’ budgets are also often challenged by an external appropriation of public funds in the form of elite capture. The term describes the practice of local leaders or bureaucrats to withdraw public funds for purposes that are unrelated to the interests of the constituency or for their private gain. It has been argued that participatory approaches to the distribution of foreign aid such as those endorsed in the World Bank’s Comprehensive Development Framework might be particularly prone to

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7 Under the supplemental Multilateral Debt Relief Initiative.
8 Instead, colonial ties and political alliances (Alesina and Dollar 2000, Harrigan, Wang and El-Said 2006) and past aid (Freytag and Pehnelt 2008) seem to be important. Freytag and Pehnelt (2008) find that this has changed with the beginning of the twenty-first century, with more focus being put on governance since then.
capture (Platteau and Gaspart 2003, Fritzen 2007). While the size and the determinants of captured funds are inherently hard to measure\(^9\), it is likely that they increase in the discretionary budget.

Apart from the literature in development economics, our model is related to political economy models on budget deficits and political fragmentation, such as in Velasco (2000) who derives a dynamic ‘deficit-spending bias’ based on the classic contribution in a static setting by Weingast, Shepsle and Johnsen (1981). In these models, socio-economic groups with self-interest compete for public funds and adopt policies of over-spending, which are individually rational but collectively inefficient. The budget exhibits the characteristics of a common pool, where decision-making groups fully internalise the benefits of the spending they propose, but not the aggregate cost. In our model, external funds share the cost of the debt burden, as external revenue increases in debt (and external spending decreases). Because the cost of external revenue is not borne by constituents (and the benefit of external spending does not accrue to them), incentives are distorted and spending becomes ‘too high’.

In the next section, we present some empirical evidence on the relation between official development assistance, debt relief and deficits and debt in developing countries during the last two decades. In Section 3, we present the dynamic optimization model of public spending in the presence of external funds as a function of the public budget. In Section 4, we provide simulations for deficits and debt for parameter values derived from data on developing countries in the 1990s and 2000s.

2. Empirical evidence on foreign aid and debt

Foreign aid is an instrument of financial support for poor countries that often exhibit a high burden of debt. Debt relief under the IMF’s and World Bank’s HIPC initiative, for example, is provided to countries that exceed a certain threshold of debt. Figures 1 and 2 below show data on net official development assistance (ODA), debt relief under the HIPC initiative, the stock of external debt (panel a) and external debt service (panel b) for 182 developing countries during 1991 to 2006. The data on

\(^{9}\) Francken, Minten and Swinnen (2008) analyze the media as determinants of capture in Madagascar.
ODA are taken from the OECD’s Development on Aid Database (2008). Data on debt relief, debt and net borrowing are obtained from the United Nations Common Database online (2008). All variables are expressed as percentages of country GDP that is also obtained from the UN database. As expected, ODA and, in particular, debt relief are positively related to debt.\textsuperscript{10} Debt service is also positively correlated with ODA and debt.\textsuperscript{11}

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\caption{Figure 1 about here}
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\begin{figure}[h]
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\caption{Figure 2 about here}
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While foreign aid is provided to countries with a high debt burden, it is also expected to support them in their efforts to achieve greater sustainability of their fiscal balance. However, the evidence points in a different direction: Figure 3 shows that the relation between ODA and net external borrowing is not negative, but positive. Our dataset does not provide enough data to look at debt relief as well, but the positive relation between debt relief and new borrowing is well documented in Easterly (2002) who derives a positive coefficient, while also controlling for national income.

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\caption{Figure 3 about here}
\end{figure}

Why is it that foreign aid apparently not only fails to free poor countries from the poverty trap but even increases their debt exposure? Easterly (2002) argues that developing countries are likely to have a high discount rate and are, therefore, inclined to compensate any debt relief with a corresponding amount of new borrowing such that the former ratio of debt to GDP is restored. This would predict a universal one-to-one increase of borrowing with debt relief. However, the effect of foreign aid and debt relief is different across countries and over time. While borrowing and debt generally seem to increase, they might do so by less than one-to-one. Easterly (2002), for example, estimates that an increase in debt relief by one percentage point of GDP increased new net borrowing by 0.34

\textsuperscript{10} The correlation coefficients are 0.59 and 0.83, respectively.
\textsuperscript{11} Correlation coefficients are 0.25 and 0.67, respectively.
percentage point of GDP in 41 heavily indebted countries during 1989-1997. In some countries borrowing has, in fact, been shown to decrease with foreign aid.\textsuperscript{12}

In the following, we develop a model of optimal control to determine the dynamically optimal amount of public spending, showing that the optimal deficit and debt are strictly positive in the presence of external funds as a function of the discretionary budget (public revenue minus debt). We also determine conditions for the deficit to increase or decrease with the share of external funds as well as the size of the increase or decrease.

\section*{3. Model of optimal public spending with external revenue}

Consider a jurisdiction with public revenue $\tau_t$, which is given and public spending $g_t$ at time $t$. The jurisdiction also receives external revenue $e_t$ at time $t$, for example in the form of foreign aid. External revenue is defined as public revenue that does not have to be financed by members of the jurisdiction (henceforth: members). The government chooses the amount of public spending $g_t$ that accrues to members, whereas $e_t$ is externally determined. We assume that a part of external revenue is proportional to public spending $\alpha_t g_t$ and can therefore be influenced by the government, whereas the other part is exogenous $\varepsilon_t$:

$$e_t = \alpha_t g_t + \varepsilon_t,$$

where we assume that $\alpha_t < 1$.\textsuperscript{13}

Exogenous external revenue cannot directly be influenced by the government in any given budget year. However, it can be affected indirectly, if it is related to the public budget. Foreign aid, for

\textsuperscript{13} Foreign aid, for example, could finance part of the spending on education or health. It is plausible to assume that $\alpha_t < 1$, such that a given amount of spending raises external revenue by less than that amount.
example, typically increases in debt, as argued in the previous section. It can also be expected to
decrease in income. Below, we therefore model external revenue as a function of public revenue net of
debt. As we will see, the presence of such revenue has repercussions on the optimal inter-temporal
spending pattern of the government.

Public spending at time $t$ can be financed either by current revenue $\tau_t$ or by borrowing at a (net) real
rate $r$, which is assumed to be constant for simplicity.\(^{14}\)

The government budget constraint in each period is the following:

$$b_t = rb_t + g_t - \tau_t - e_t,$$

where $b_t$ is the stock of debt at time $t$. Note that external revenue $e_t > 0$ decreases the deficit $b_t$, ceteris paribus.

Public spending $g_t$ is expressed as a share of public revenue net of debt service at any given time $t$, $\phi_{gt} > 0$. Exogenous external revenue $e_t$ is expressed as a share of debt minus public revenue, $\phi_{et} > 0$:

$$g_t = \phi_{gt}(\tau_t - rb_t),$$

$$e_t = \phi_{et}(b_t - \tau_t).$$

According to (1.4), we have external revenue $e_t > 0$ for $b_t > \tau_t$, which increases in debt and
decreases in public revenue. The opposite is true for external spending $e_t < 0$ in the case of $b_t < \tau_t$.\(^{15}\)

\(^{14}\) Note that, for the results to follow, it does not matter whether we keep revenue fixed and have the government
decide on the budget via spending or vice versa.
In the following, we will focus on external revenue ($\alpha > 0$ and $\epsilon > 0$). All results carry through analogously for external spending.

From (1.1) together with (1.3) and (1.4), it follows that total external revenue equals

$$e_t = \alpha_t \phi_{st}(\tau_t - r b_t) + \phi_{st}(b_t - \tau_t), \tag{1.5}$$

The share of external revenue $\phi_{st}$ and $\tau_t$ follow a given time path, with $\epsilon$ being determined by $b_t$ according to (1.4). $\phi_{st}$ is the control variable, to be chosen optimally by the government while taking into account its effect on the state variable of public debt, $b_t$, which in turn affects the budget constraint.

Assuming that debt cannot grow indefinitely, we impose the common ‘no-Ponzi-game rule’:

$$\lim_{t \to \infty} b_t e^{-rt} \leq 0. \tag{1.6}$$

The government chooses the share of public net revenue to be spent on members, $\phi_{st}$, in order to maximise their utility. $\text{Utility is given by the present value of the natural log of total spending:}$

$$U = \int_{t=0}^{\infty} \ln g_t e^{-rt} dt. \tag{1.7}$$

An appropriation of public funds via capture, for example, is likely to increase in public revenue and decrease in debt, as argued in the introduction. It is easy to allow for external spending to change to a different extent with a change in public revenue or debt by adding a factor to $\tau_t$ or $b_t$ in (1.4). This would not qualitatively change results.

Note that debt relief, for example, is typically only given after the ratio of debt to GDP exceeds a certain threshold $\overline{b}$. In this case, an exogenously given time path for $\phi_{st}$ could consist of $\phi_{st} = 0$ for $b_t < \overline{b}$ and $\phi_{st} > 0$ for $b_t > \overline{b}$.

We assume that members of the constituency are identical, abstracting from intra-governmental redistributive issues.

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It is maximized subject to (1.1) – (1.4) and (1.6) and to a given initial level of debt \( b_0 \). We assume that \( b_0 = 0 \) without loss of generality. In (1.6), it is assumed that the time preference rate is equal to the interest rate \( r \). This way, if revenue \( \tau_t \) is constant over time, there are no transfer-smoothing reasons for debt accumulation. When choosing spending \( g_t \) via its choice of \( \phi_{gt} \), the government has to take into account that by increasing current spending it also decreases the scope for future spending, which is restricted via the budget constraint (1.2).

Solving this optimal control problem gives the optimal share of spending at any given time \( t \):

\[
\phi_{gt} = \frac{1}{1 - \alpha_t} \left[ 1 + \phi_{gt} \left( \frac{b_t - \tau_t}{\tau_t - rb_t} + \frac{1}{r} \right) \right].
\]  

(1.8)

Optimal spending increases in \( \alpha_t \), as external financing decreases the cost of spending proportionally. It also increases in the share of exogenous external revenue \( \phi_{et} \).

Next, we determine the size of public debt that results from this spending rule. Substituting (1.8) into (1.2) using (1.3) and (1.5) and solving the differential equation gives the endogenous public debt at time \( t \):

\[
b_t = \frac{\tau_t}{r} \left( 1 - e^{-\phi_{et}t} \right).
\]  

(1.9)

Substituting (1.9) in (1.2), using (1.3), (1.5) and (1.8)\(^{19}\), gives the change in public debt over time, i.e. the budget deficit:

\[
\dot{b}_t = \frac{\tau_t}{r} \phi_{et} e^{-\phi_{et}t}.
\]  

(1.10)

\(^{18}\) See Appendix B for details.

\(^{19}\) Or, equivalently, differentiating (1.9) with respect to time \( t \).
From (1.9) and (1.10), we immediately find

**Proposition 1:** In the presence of exogenous external revenue \( \phi_\text{ex} > 0 \), it is optimal for the government to run a deficit and to accumulate debt. If \( \phi_\text{ex} = 0 \), the government chooses a balanced budget. The deficit and debt are independent of endogenous external revenue.

In the absence of exogenous external revenue, it is optimal for the government to have a balanced budget and to incur no debt. This is the equivalent of zero optimal debt in the tax-smoothing model of Barro (1979).\(^{20}\) If, however, some part of the budget consists of external revenue that is increasing in debt, it becomes optimal for the government to run a deficit. The government chooses to increase spending by more than the amount of extra revenue received and bear the corresponding interest burden rather than to increase spending only proportionally and keep the budget balanced. To see why, consider the case without external revenue. There, the government opts for a balanced budget, because the benefit of higher current spending comes at the cost of an even higher debt service. With external revenue, however, spending comes at a cost that is lower. Since external revenue is an increasing function of debt, the government can increase future external revenue by increasing the current deficit. As a consequence, the optimal budget deficit rises to some value above zero.

In contrast, the government increases the spending share with an increase in the proportion of endogenous external revenue \( \alpha_t \) (compare (1.8)) only to the extent that the budget remains balanced, given that \( \phi_\text{ex} = 0 \). For example for \( \alpha_t = 0.5 \), the spending share increases to \( \phi_g = 2 \), up from \( \phi_g = 1 \) for \( \alpha_t = 0 \), such that the total share of the discretionary budget spent \( (1-\alpha_t)\phi_g \) remains equal to 1.

\(^{20}\) There, deficits become optimal in the presence of unanticipated, temporary shocks to revenue or spending.
Since the optimal deficit is positive in the presence of exogenous external spending, public debt is positive and increasing in $\phi_{et}$ (see 1.9). Like the budget deficit, public debt is independent of the factor $\alpha_t$ of endogenous external spending.

So far, we have derived the optimal time paths of spending, the deficit and debt in the presence of external revenue. Next, we look at the effects of a change in external revenue. The effect of foreign aid on the deficit is the subject of a comprehensive literature in development economics (recent studies for example by Ouattara 2006 and Feeny 2007), but findings vary. This literature typically develops and estimates some form of fiscal response model according to which the government minimizes deviations of fiscal policy variables from some pre-specified target values subject to a budget constraint including foreign aid disbursements. However, these models are static because they do not allow for any debt to accumulate. Our model provides a dynamic framework for spending decisions and the associated time paths of the deficit and debt in the presence of external revenue such as foreign aid. Within this framework, we find that the effect of foreign aid on the deficit is path-dependent and can be either negative or positive:

**Proposition 2:** Assume that $\phi_{et}$ is constant over time. An increase in the share of exogenous external funds $\phi_e$ increases the deficit, if $\phi_e < 1 / t$. It decreases the deficit, if $\phi_e > 1 / t$.

The optimal deficit increases in $\phi_e$, as long as $\phi_e$ and $t$ are not too large. While a deficit can increase the utility of members by generating external revenue, the resulting interest burden of debt service can grow too large to make a policy of increasing deficits optimal. At any given point in time, deficits decline the sooner, the higher $\phi_e$, ceteris paribus. Further, deficits decline the sooner, the further along the system is in time, converging to $\dot{b}_t = 0$ and to $b_t = \frac{r}{r'}$ for $t = \infty$ independent of the value...

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21 The higher $\phi_e$ is, the higher is public debt - see (1.9).
of \( \phi_e \). External revenue therefore causes transitional deviations of fiscal variables, but it does not have an effect on their steady states.

The convergence path of the deficit to its steady state also depends on public revenue \( \tau \) and on the interest rate. From (1.10), we find

**Proposition 3: Assume that \( \tau \) is constant over time. Given \( \phi_{et} > 0 \), an increase in public revenue \( \tau \) increases the deficit.**

An increase in public revenue decreases the amount \( \epsilon \) that is externally financed, for any given \( \phi_{et} \). By increasing the deficit, the government can increase external revenue back to the optimum.\(^{22}\) In contrast, the tax-smoothing rule would require keeping the budget balanced in case of a permanent change in revenue.

**Proposition 4: Given \( \phi_{et} > 0 \), an increase in the interest rate \( r \) decreases the deficit.**

An increase in the interest rate increases the amount \( \epsilon \) that is externally financed, for any given \( \phi_{et} \). As deficit-spending becomes more expensive, the government decreases the deficit and decreases external revenue back to the optimum. Again, this is in contrast to tax-smoothing, which would suggest keeping the budget balanced in case of a permanent change in the interest rate.

In the following, we provide a sense for the magnitude of the effects described above by deriving simulations of the optimal deficit for a range of parameter values for the interest rate, public revenue and the share of external revenue \( \phi_e \). We then use existing data on foreign aid and fiscal policy variables in developing countries to compare our simulated findings with real-world data.

\(^{22}\) There is empirical evidence for developing countries to borrow more during good times (in contrast to developed countries) in Aguiar and Gopinath (2007).
4. Simulations

For an illustration of the results derived above, we quantitatively assess the optimal values of the budget deficit (in per cent of GDP) for a range of plausible values of $\phi_{et}$, the interest rate $r$ and income $\tau$ (in per cent of GDP), which we assume to be constant over time. Data on foreign aid and fiscal policy in developing countries are available from the OECD’s Development on Aid Database (2008) and the United Nations Common Database (2008). We use an unbalanced panel of 182 developing countries during 1991 to 2006. Foreign aid in the form of official development assistance (ODA) amounts to an average of 5 per cent of GDP in our sample. Debt relief under the HIPC initiative averages 44 per cent of GDP. Public revenue data are not readily available for many developing countries. According to the data presented in Gordon and Li (2005), Table 2, average tax revenue was 17 per cent of GDP in 26 developing countries during 1996 – 2001. Average debt in our sample is 81 per cent of GDP, and net borrowing is 1.5 per cent of GDP. External long-term debt service averages 4 per cent of GDP.

In Tables 1 and 2, we report simulation results for the optimal budget deficit in per cent of GDP. Computing $\phi_{et}$ as the ratio of ODA to debt net of tax revenue (see (1.4)) from the data in our sample, we find an average value of 0.02. In the simulation, we use values of $\phi_{et}$ between 0 and 0.04 as a plausible range. For public revenue $\tau$, we use values between 20 and 30 per cent of GDP, and for the interest rate $r$ we choose a range of 5-15 per cent. Since optimal values of the deficit are path-dependent, we also need to choose an initial value of debt and a specific point in time for our simulation. We assume an initial debt value $b_0$ of 100 per cent of GDP and choose $t = 1$ for a system that is at the beginning of the convergence path.

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23 The measure $\tau$ of public revenue in theory also comprises public revenue from other sources such as tariffs or revenue from the sale of publicly produced goods, data on which are even scarcer.

24 Outliers with net borrowing of more than 100 per cent of GDP or less than 100 per cent of GDP (12 observations) are not included.

25 Average debt was 95 per cent of GDP in 1991, the first year of our sample.

26 For $b_0 > 0$, the deficit at time $t$ is given by $\delta_t = \phi_{et} \left( \frac{\tau_t - b_0}{r} \right) e^{-r t}$. 
Table 1 shows, for example, that the optimal deficit is 1.96 per cent of GDP for an exogenous revenue share $\phi_{rt}$ of 0.02 when the interest rate is 10 per cent and the share of public revenue in GDP is 20 per cent. The optimal deficit increases to 3.84 per cent for $\phi_{rt}$ of 0.04. The deficit decreases in the interest rate and increases in public revenue. The average deficit of 1.5 per cent in our sample together with an average interest rate of 5 per cent and public revenue of 20 per cent of GDP would be consistent with an exogenous external funding share of 0.005, $t=1$ and 100 per cent of initial debt according to (1.10).

In Table 2, we report results on the optimal deficit over time, for $\tau = 20$ per cent of GDP. As before, we assume that initial debt $b_0$ is 100 per cent of GDP. We can see that the deficit decreases in time at a decreasing rate.

5. Conclusion

Debt in developing countries has kept growing despite increases in foreign aid in recent decades. To the extent that foreign flows are positively related to indebtedness, they can be expected to give rise to moral hazard and have adverse effects on fiscal sustainability. For example, policies such as the HIPC initiative by the IMF and the World Bank grant debt relief to countries whose debt exceeds a certain threshold. Data on official development assistance during 1991-2006 also confirm a positive correlation of foreign aid with receiving countries’ level of debt.

We analyze optimal borrowing and the corresponding development of debt in a dynamic framework with external revenue that is contingent on the level of debt. We find that the optimal deficit is positive in the presence of external revenue that decreases in income and increases in debt. The result applies
in the same way to external spending that increases in income and decreases in debt. Foreign aid or the capture of public funds by local elites can therefore be expected to induce incentives for deficit-spending and, as a consequence, raise public debt.

Within our framework, we can also show that optimal deficits can either increase or decrease in the size of external revenue, depending on time since the beginning of the convergence of public debt to its steady state. Further, we show that optimal deficits increase in public revenue and decrease in the interest rate for any given positive share of exogenous external revenue at any given time.

Our findings suggest that foreign aid in its current form can be detrimental to fiscal sustainability in receiving countries. In order to improve incentives for deficit spending, the distribution of foreign funds should not be contingent on debt levels. Instead, it could finance a share of given spending, for example on public services that are conducive to growth such as education and health, or be based on measures of outcome, such as improvements in debt management. Recent attempts to strengthen the conditionality of foreign funds on good governance should also address the issue of moral hazard that is inherent in current allocation practices, reinforcing unsustainable fiscal behaviour.
Figures

Figure 1. Official development assistance (ODA) and debt in developing countries, 1991 – 2006.
Figure 2. Debt relief under HIPC initiative and debt in developing countries, 1991 – 2006.

(a)

(b)
Figure 3. ODA and borrowing, 1991 – 2006.

Note: Outliers with net borrowing smaller than -100 or larger than 100 per cent of GDP are not shown.
Table 1. Simulated optimal deficit (per cent of GDP) at t=1, $b_0=100$ (per cent of GDP).

Baseline parameterisation: $\tau = 20$ (per cent of GDP).

<table>
<thead>
<tr>
<th>Interest rate $r$</th>
<th>$\phi_{\tau t} = 0$</th>
<th>$\phi_{\tau t} = 0.01$</th>
<th>$\phi_{\tau t} = 0.02$</th>
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Increase in $\tau$: $\tau = 25$ (per cent of GDP).

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<th>Interest rate $r$</th>
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<td>0.05</td>
<td>0</td>
<td>3.96</td>
<td>7.84</td>
<td>11.64</td>
<td>15.37</td>
</tr>
<tr>
<td>0.10</td>
<td>0</td>
<td>1.48</td>
<td>2.94</td>
<td>4.36</td>
<td>5.76</td>
</tr>
<tr>
<td>0.15</td>
<td>0</td>
<td>0.66</td>
<td>1.30</td>
<td>1.94</td>
<td>2.56</td>
</tr>
</tbody>
</table>

Increase in $\tau$: $\tau = 30$ (per cent of GDP).

<table>
<thead>
<tr>
<th>Interest rate $r$</th>
<th>$\phi_{\tau t} = 0$</th>
<th>$\phi_{\tau t} = 0.01$</th>
<th>$\phi_{\tau t} = 0.02$</th>
<th>$\phi_{\tau t} = 0.03$</th>
<th>$\phi_{\tau t} = 0.04$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0</td>
<td>4.95</td>
<td>9.80</td>
<td>14.55</td>
<td>19.21</td>
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<tr>
<td>0.10</td>
<td>0</td>
<td>1.98</td>
<td>3.92</td>
<td>5.82</td>
<td>7.68</td>
</tr>
<tr>
<td>0.15</td>
<td>0</td>
<td>0.99</td>
<td>1.96</td>
<td>2.91</td>
<td>3.84</td>
</tr>
</tbody>
</table>

Table 2. Simulated optimal deficit (per cent of GDP) over time, $b_0=100$ (per cent of GDP) and $\tau = 20$ (per cent of GDP).

Stage: t=1.

<table>
<thead>
<tr>
<th>Interest rate $r$</th>
<th>$\phi_{\tau t} = 0$</th>
<th>$\phi_{\tau t} = 0.01$</th>
<th>$\phi_{\tau t} = 0.02$</th>
<th>$\phi_{\tau t} = 0.03$</th>
<th>$\phi_{\tau t} = 0.04$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0</td>
<td>2.97</td>
<td>5.88</td>
<td>8.73</td>
<td>11.52</td>
</tr>
<tr>
<td>0.10</td>
<td>0</td>
<td>0.99</td>
<td>1.96</td>
<td>2.91</td>
<td>3.84</td>
</tr>
<tr>
<td>0.15</td>
<td>0</td>
<td>0.33</td>
<td>0.65</td>
<td>0.97</td>
<td>1.28</td>
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</tbody>
</table>

Stage: t=5.

<table>
<thead>
<tr>
<th>Interest rate $r$</th>
<th>$\phi_{\tau t} = 0$</th>
<th>$\phi_{\tau t} = 0.01$</th>
<th>$\phi_{\tau t} = 0.02$</th>
<th>$\phi_{\tau t} = 0.03$</th>
<th>$\phi_{\tau t} = 0.04$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0</td>
<td>2.85</td>
<td>5.42</td>
<td>7.74</td>
<td>9.82</td>
</tr>
<tr>
<td>0.10</td>
<td>0</td>
<td>0.95</td>
<td>1.80</td>
<td>2.58</td>
<td>3.27</td>
</tr>
<tr>
<td>0.15</td>
<td>0</td>
<td>0.31</td>
<td>0.60</td>
<td>0.86</td>
<td>1.09</td>
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</table>

Stage: t=9.

<table>
<thead>
<tr>
<th>Interest rate $r$</th>
<th>$\phi_{\tau t} = 0$</th>
<th>$\phi_{\tau t} = 0.01$</th>
<th>$\phi_{\tau t} = 0.02$</th>
<th>$\phi_{\tau t} = 0.03$</th>
<th>$\phi_{\tau t} = 0.04$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0</td>
<td>2.74</td>
<td>5.01</td>
<td>6.87</td>
<td>8.37</td>
</tr>
<tr>
<td>0.10</td>
<td>0</td>
<td>0.91</td>
<td>1.67</td>
<td>2.29</td>
<td>2.79</td>
</tr>
<tr>
<td>0.15</td>
<td>0</td>
<td>0.30</td>
<td>0.55</td>
<td>0.76</td>
<td>0.93</td>
</tr>
</tbody>
</table>
## Appendix A – Variable Description and Sources

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean&lt;sup&gt;a&lt;/sup&gt;</th>
<th>S.D. &lt;sup&gt;a&lt;/sup&gt;</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget deficit (in per cent of GDP)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>1.54</td>
<td>8.17</td>
<td>Net borrowing (+)/net lending (-) of the nation [code 30239] x Exchange rate</td>
<td>United Nations Common Database (2008)</td>
</tr>
<tr>
<td>Official development assistance (ODA) (in per cent of GDP)</td>
<td>5.56</td>
<td>9.43</td>
<td>ODA total, net disbursements, current prices (USD), DAC countries</td>
<td>OECD Development on Aid Database (2008)</td>
</tr>
<tr>
<td>Debt relief (in per cent of GDP)</td>
<td>44.40</td>
<td>52.21</td>
<td>Debt relief committed under HIPC initiative, cumulative US$ million (IMF/MDG) [code 30009]</td>
<td>United Nations Common Database (2008)</td>
</tr>
<tr>
<td>Tax revenue (in per cent of GDP)</td>
<td>17.69</td>
<td>4.25</td>
<td>Tax revenue (1996-2001)</td>
<td>Table 2 in Gordon and Li (2005)</td>
</tr>
</tbody>
</table>

<sup>a</sup> Unbalanced panel of 182 developing countries during 1991-2006 (except for tax revenue as mentioned).

<sup>b</sup> Excluding outliers with values larger than 100 or smaller than -100 per cent of GDP (12 observations).
Appendix B – Optimal control problem

The Hamiltonian corresponding to our optimal control problem is the following:

\[ H = e^{-rt} \ln \left[ \phi_g (\tau_t - rb_t) \right] + p_t \left[ (\tau_t - rb_t)(1 + (1 - \alpha)\phi_g) - \phi_{rt} (b_t - \tau_t) \right]. \]  

(A.1)

The maximum principle requires that

\[ \frac{\partial H}{\partial \phi_g} = 0, \]  

(A.2)

\[ \frac{\partial H}{\partial p_t} = \dot{b}_t, \]  

(A.3)

\[ \frac{\partial H}{\partial b_t} = -\dot{p}_t. \]  

(A.4)

It follows that the time paths of the control variable (the share of spending on members \( \phi_g \)), the state variable (public debt \( b_t \)) and the value of public debt \( p_t \) are jointly determined by the following three formulas:

\[ e^{-rt} + p_t (1 - \alpha)(\tau_t - rb_t) = 0, \]  

(A.2’)

\[ \dot{b}_t = (\tau_t - rb_t)(1 + (1 - \alpha)\phi_g) - \phi_{rt} (b_t - \tau_t), \]  

(A.3’)

\[ \dot{p}_t = \frac{r e^{-rt}}{\tau_t - rb_t} + p_t \left[ r(-1 + (1 - \alpha)\phi_g) + \phi_{rt} \right]. \]  

(A.4’)

To solve these three equations for the three unknowns, we first express (A.2’) as a function of \( p_t \):

\[ p_t = -\frac{e^{-rt}}{\phi_g (1 - \alpha)(\tau_t - rb_t)}. \]  

(A.2’’)
We then substitute for $p_t$ in (A.4') using (A.2'') and set the resulting expression for $\dot{p}_t$ equal to the first derivative of (A.2'') with respect to time, using (A.3') for the derivative of $b_t$.

Solving for the control variable $\phi_{et}$, we get the following optimal value:

$$\phi_{et} = \frac{1}{1 - \alpha_i} \left[ 1 + \phi_{ct} \left( \frac{b_t - \tau_i}{\tau_i - rb_t} + \frac{1}{r} \right) \right], \quad (A.5)$$

which results in optimal values for public debt

$$b_t = \frac{\tau_i}{r} \left( 1 - e^{-\phi_{et}} \right) \quad (A.6)$$

and the deficit

$$\dot{b}_t = \frac{\tau_i}{r} \phi_{et} e^{-\phi_{et}}, \quad (A.7)$$

which is positive for $\phi_{et} > 0$. 


References


Gordon, R. and W. Li (2005), Tax Structures in Developing Countries: Many Puzzles and a Possible Explanation, Department of Economics, UC San Diego.


OECD Development Database on Aid from DAC Members: DAC online, [http://www.oecd.org/document/33/0,2340,en_2649_34447_36661793_1_1_1_1,00.html](http://www.oecd.org/document/33/0,2340,en_2649_34447_36661793_1_1_1_1,00.html) as of July 2008.


